Chapter F
Foundations: A Prelude to Functions

Section F.1

1. 0

2. $|5-(-3)| = |8| = 8$

3. $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$

4. $11^2 + 60^2 = 121 + 3600 = 3721 = 61^2$
Since the sum of the squares of two of the sides of the triangle equals the square of the third side, the triangle is a right triangle.

5. x-coordinate, or abscissa; y-coordinate, or ordinate.

6. quadrants

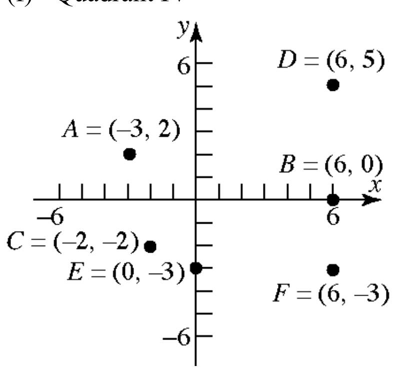
7. midpoint

8. False; the distance between two points is never negative.

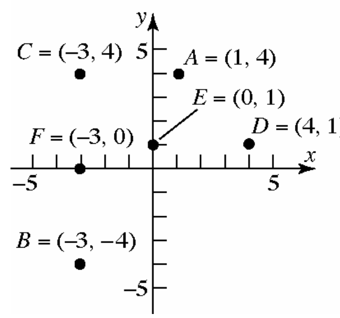
9. False; points that lie in Quadrant IV will have a positive x-coordinate and a negative y-coordinate. The point $(-1, 4)$ lies in Quadrant II.

10. True; $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

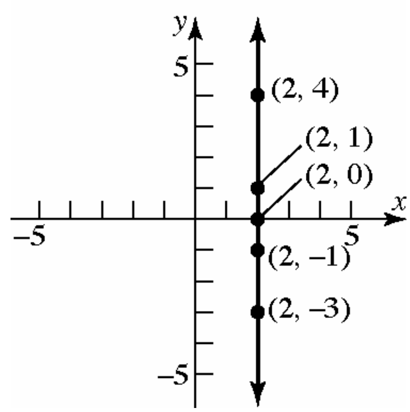
11. (a) Quadrant II
(b) Positive x-axis
(c) Quadrant III
(d) Quadrant I
(e) Negative y-axis
(f) Quadrant IV



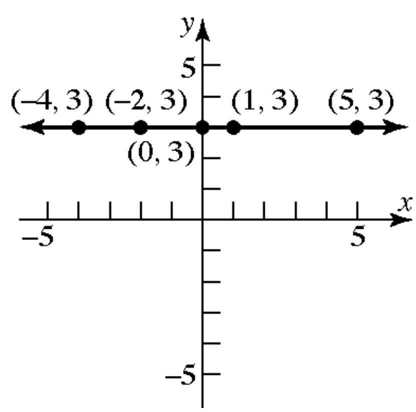
12. (a) Quadrant I
(b) Quadrant III
(c) Quadrant II
(d) Quadrant I
(e) Positive y-axis
(f) Negative x-axis



13. The points will be on a vertical line that is two units to the right of the y-axis.



14. The points will be on a horizontal line that is three units above the x-axis.



15. $d(P_1,P_2)=\sqrt{(2-0)^2+(1-0)^2}=\sqrt{4+1}=\sqrt{5}$

16. $d(P_1,P_2)=\sqrt{(-2-0)^2+(1-0)^2}=\sqrt{4+1}=\sqrt{5}$

17. $d(P_1,P_2)=\sqrt{(-2-1)^2+(2-1)^2}=\sqrt{9+1}=\sqrt{10}$

18. $d(P_1,P_2)=\sqrt{\left(2-(-1)\right)^2+(2-1)^2}$
$=\sqrt{9+1}=\sqrt{10}$

19. $d(P_1,P_2)=\sqrt{(5-3)^2+\left(4-(-4)\right)^2}=\sqrt{2^2+(8)^2}$
$=\sqrt{4+64}=\sqrt{68}=2\sqrt{17}$

20. $d(P_1,P_2)=\sqrt{\left(2-(-1)\right)^2+(4-0)^2}=\sqrt{(3)^2+4^2}$
$=\sqrt{9+16}=\sqrt{25}=5$

21. $d(P_1,P_2)=\sqrt{\left(6-(-3)\right)^2+(0-2)^2}$
$=\sqrt{9^2+(-2)^2}=\sqrt{81+4}=\sqrt{85}$

22. $d(P_1,P_2)=\sqrt{(4-2)^2+\left(2-(-3)\right)^2}=\sqrt{2^2+5^2}$
$=\sqrt{4+25}=\sqrt{29}$

23. $d(P_1,P_2)=\sqrt{(6-4)^2+\left(4-(-3)\right)^2}=\sqrt{2^2+7^2}$
$=\sqrt{4+49}=\sqrt{53}$

24. $d(P_1,P_2)=\sqrt{\left(6-(-4)\right)^2+\left(2-(-3)\right)^2}$
$=\sqrt{10^2+5^2}=\sqrt{100+25}$
$=\sqrt{125}=5\sqrt{5}$

25. $d(P_1,P_2)=\sqrt{\left(2.3-(-0.2\right)^2+(1.1-0.3)^2}$
$=\sqrt{(2.5)^2+(0.8)^2}=\sqrt{6.25+0.64}$
$=\sqrt{6.89}\approx 2.62$

26. $d(P_1,P_2)=\sqrt{\left(-0.3-1.2\right)^2+(1.1-2.3)^2}$
$=\sqrt{(-1.5)^2+(-1.2)^2}=\sqrt{2.25+1.44}$
$=\sqrt{3.69}\approx 1.92$

27. $d(P_1,P_2)=\sqrt{(0-a)^2+(0-b)^2}=\sqrt{a^2+b^2}$

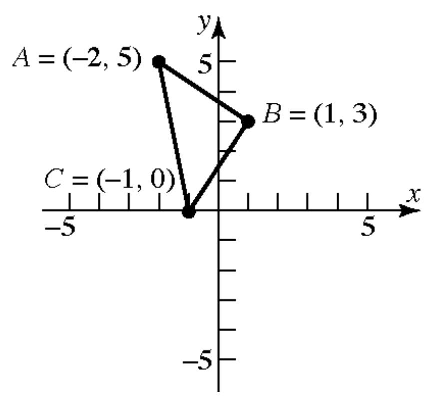
28. $d(P_1,P_2)=\sqrt{(0-a)^2+(0-a)^2}=\sqrt{a^2+a^2}$
$=\sqrt{2a^2}=\sqrt{2}\,|\,a\,|$

29. $A=(-2,5),\ B=(1,3),\ C=(-1,0)$

$d(A,B)=\sqrt{\left(1-(-2)\right)^2+(3-5)^2}$
$=\sqrt{3^2+(-2)^2}=\sqrt{9+4}$
$=\sqrt{13}$

$d(B,C)=\sqrt{\left(-1-1\right)^2+(0-3)^2}$
$=\sqrt{(-2)^2+(-3)^2}=\sqrt{4+9}$
$=\sqrt{13}$

$d(A,C)=\sqrt{\left(-1-(-2)\right)^2+(0-5)^2}$
$=\sqrt{1^2+(-5)^2}=\sqrt{1+25}$
$=\sqrt{26}$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$\left[d(A,B)\right]^2+\left[d(B,C)\right]^2=\left[d(A,C)\right]^2$$
$$\left(\sqrt{13}\right)^2+\left(\sqrt{13}\right)^2=\left(\sqrt{26}\right)^2$$
$$13+13=26$$
$$26=26$$

The area of a triangle is $A=\frac{1}{2}\cdot bh$. In this problem,

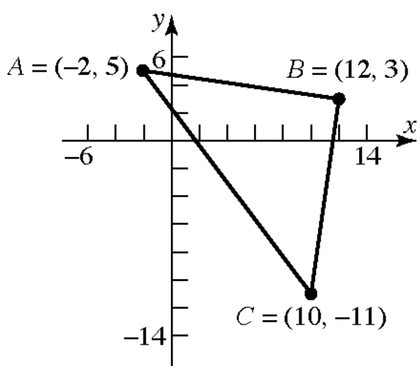
$$A=\frac{1}{2}\cdot\left[d(A,B)\right]\cdot\left[d(B,C)\right]$$
$$=\frac{1}{2}.\sqrt{13}\cdot\sqrt{13}=\frac{1}{2}\cdot 13$$
$$=\frac{13}{2}\text{ square units}$$

30. $A=(-2,5),\ B=(12,3),\ C=(10,-11)$

$$\begin{aligned} d(A,B) &= \sqrt{(12-(-2))^2+(3-5)^2} \\ &= \sqrt{14^2+(-2)^2} \\ &= \sqrt{196+4} = \sqrt{200} \\ &= 10\sqrt{2} \end{aligned}$$

$$\begin{aligned} d(B,C) &= \sqrt{(10-12)^2+(-11-3)^2} \\ &= \sqrt{(-2)^2+(-14)^2} \\ &= \sqrt{4+196} = \sqrt{200} \\ &= 10\sqrt{2} \end{aligned}$$

$$\begin{aligned} d(A,C) &= \sqrt{(10-(-2))^2+(-11-5)^2} \\ &= \sqrt{12^2+(-16)^2} \\ &= \sqrt{144+256} = \sqrt{400} \\ &= 20 \end{aligned}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$\begin{aligned} [d(A,B)]^2+[d(B,C)]^2 &= [d(A,C)]^2 \\ \left(10\sqrt{2}\right)^2+\left(10\sqrt{2}\right)^2 &= (20)^2 \\ 200+200 &= 400 \\ 400 &= 400 \end{aligned}$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

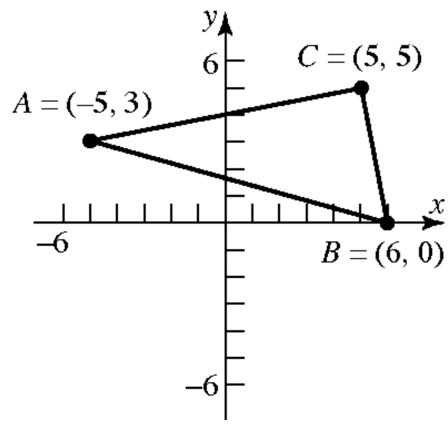
$$\begin{aligned} A &= \frac{1}{2}\cdot[d(A,B)]\cdot[d(B,C)] \\ &= \frac{1}{2}\cdot 10\sqrt{2}\cdot 10\sqrt{2} \\ &= \frac{1}{2}\cdot 100\cdot 2 \\ &= 100 \text{ square units} \end{aligned}$$

31. $A=(-5,3),\ B=(6,0),\ C=(5,5)$

$$\begin{aligned} d(A,B) &= \sqrt{(6-(-5))^2+(0-3)^2} \\ &= \sqrt{11^2+(-3)^2} = \sqrt{121+9} \\ &= \sqrt{130} \end{aligned}$$

$$\begin{aligned} d(B,C) &= \sqrt{(5-6)^2+(5-0)^2} \\ &= \sqrt{(-1)^2+5^2} = \sqrt{1+25} \\ &= \sqrt{26} \end{aligned}$$

$$\begin{aligned} d(A,C) &= \sqrt{(5-(-5))^2+(5-3)^2} \\ &= \sqrt{10^2+2^2} = \sqrt{100+4} \\ &= \sqrt{104} \\ &= 2\sqrt{26} \end{aligned}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$\begin{aligned} [d(A,C)]^2+[d(B,C)]^2 &= [d(A,B)]^2 \\ \left(\sqrt{104}\right)^2+\left(\sqrt{26}\right)^2 &= \left(\sqrt{130}\right)^2 \\ 104+26 &= 130 \\ 130 &= 130 \end{aligned}$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

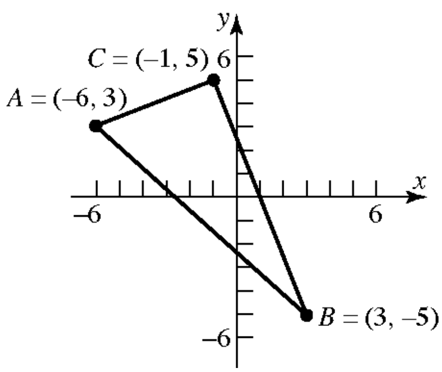
$$\begin{aligned} A &= \frac{1}{2}\cdot[d(A,C)]\cdot[d(B,C)] \\ &= \frac{1}{2}\cdot\sqrt{104}\cdot\sqrt{26} \\ &= \frac{1}{2}\cdot 2\sqrt{26}\cdot\sqrt{26} \\ &= \frac{1}{2}\cdot 2\cdot 26 \\ &= 26 \text{ square units} \end{aligned}$$

32. $A = (-6, 3),\ B = (3, -5),\ C = (-1, 5)$

$$d(A,B) = \sqrt{\left(3-(-6)\right)^2 + (-5-3)^2} = \sqrt{9^2 + (-8)^2} = \sqrt{81+64} = \sqrt{145}$$

$$d(B,C) = \sqrt{\left(-1-3\right)^2 + (5-(-5))^2} = \sqrt{(-4)^2 + 10^2} = \sqrt{16+100} = \sqrt{116} = 2\sqrt{29}$$

$$d(A,C) = \sqrt{\left(-1-(-6)\right)^2 + (5-3)^2} = \sqrt{5^2 + 2^2} = \sqrt{25+4} = \sqrt{29}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$[d(A,C)]^2 + [d(B,C)]^2 = [d(A,B)]^2$$
$$\left(\sqrt{29}\right)^2 + \left(\sqrt{116}\right)^2 = \left(\sqrt{145}\right)^2$$
$$29 + 116 = 145$$
$$145 = 145$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

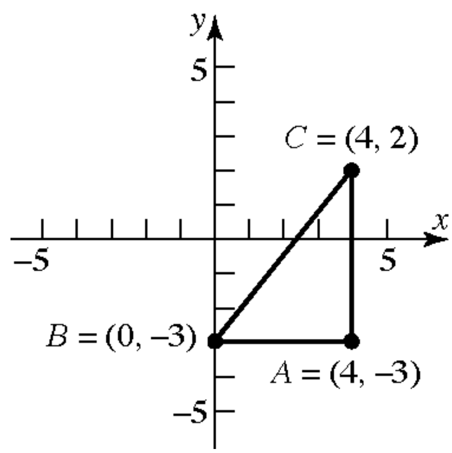
$$A = \frac{1}{2}\cdot[d(A,C)]\cdot[d(B,C)] = \frac{1}{2}\cdot\sqrt{29}\cdot\sqrt{116} = \frac{1}{2}\cdot\sqrt{29}\cdot 2\sqrt{29} = \frac{1}{2}\cdot 2\cdot 29 = 29 \text{ square units}$$

33. $A = (4,-3),\ B = (0,-3),\ C = (4,2)$

$$d(A,B) = \sqrt{(0-4)^2 + \left(-3-(-3)\right)^2} = \sqrt{(-4)^2 + 0^2} = \sqrt{16+0} = \sqrt{16} = 4$$

$$d(B,C) = \sqrt{\left(4-0\right)^2 + \left(2-(-3)\right)^2} = \sqrt{4^2 + 5^2} = \sqrt{16+25} = \sqrt{41}$$

$$d(A,C) = \sqrt{(4-4)^2 + \left(2-(-3)\right)^2} = \sqrt{0^2 + 5^2} = \sqrt{0+25} = \sqrt{25} = 5$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$[d(A,B)]^2 + [d(A,C)]^2 = [d(B,C)]^2$$
$$4^2 + 5^2 = \left(\sqrt{41}\right)^2$$
$$16 + 25 = 41$$
$$41 = 41$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

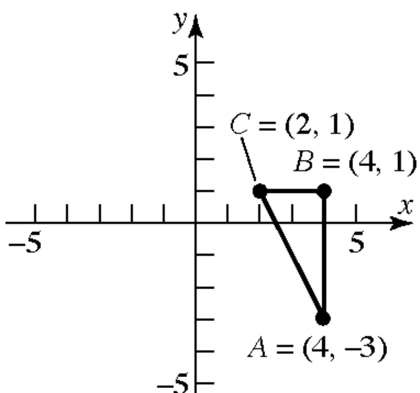
$$A = \frac{1}{2}\cdot[d(A,B)]\cdot[d(A,C)] = \frac{1}{2}\cdot 4\cdot 5 = 10 \text{ square units}$$

34. $A = (4, -3),\ B = (4, 1),\ C = (2, 1)$

$$\begin{aligned} d(A,B) &= \sqrt{(4-4)^2 + (1-(-3))^2} \\ &= \sqrt{0^2 + 4^2} \\ &= \sqrt{0+16} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

$$\begin{aligned} d(B,C) &= \sqrt{(2-4)^2 + (1-1)^2} \\ &= \sqrt{(-2)^2 + 0^2} = \sqrt{4+0} \\ &= \sqrt{4} \\ &= 2 \end{aligned}$$

$$\begin{aligned} d(A,C) &= \sqrt{(2-4)^2 + (1-(-3))^2} \\ &= \sqrt{(-2)^2 + 4^2} = \sqrt{4+16} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$\begin{aligned} [d(A,B)]^2 + [d(B,C)]^2 &= [d(A,C)]^2 \\ 4^2 + 2^2 &= \left(2\sqrt{5}\right)^2 \\ 16 + 4 &= 20 \\ 20 &= 20 \end{aligned}$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$\begin{aligned} A &= \frac{1}{2}\cdot[d(A,B)]\cdot[d(B,C)] \\ &= \frac{1}{2}\cdot 4\cdot 2 \\ &= 4 \text{ square units} \end{aligned}$$

35. The coordinates of the midpoint are:

$$\begin{aligned} (x,y) &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{3+5}{2}, \frac{-4+4}{2}\right) \\ &= \left(\frac{8}{2}, \frac{0}{2}\right) \\ &= (4, 0) \end{aligned}$$

36. The coordinates of the midpoint are:

$$\begin{aligned} (x,y) &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{-2+2}{2}, \frac{0+4}{2}\right) \\ &= \left(\frac{0}{2}, \frac{4}{2}\right) \\ &= (0, 2) \end{aligned}$$

37. The coordinates of the midpoint are:

$$\begin{aligned} (x,y) &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{-3+6}{2}, \frac{2+0}{2}\right) \\ &= \left(\frac{3}{2}, \frac{2}{2}\right) \\ &= \left(\frac{3}{2}, 1\right) \end{aligned}$$

38. The coordinates of the midpoint are:

$$\begin{aligned} (x,y) &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{2+4}{2}, \frac{-3+2}{2}\right) \\ &= \left(\frac{6}{2}, \frac{-1}{2}\right) \\ &= \left(3, -\frac{1}{2}\right) \end{aligned}$$

39. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\\ &= \left(\frac{4+6}{2}, \frac{-3+1}{2}\right)\\ &= \left(\frac{10}{2}, \frac{-2}{2}\right)\\ &= (5, -1)\end{aligned}$$

40. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\\ &= \left(\frac{-4+2}{2}, \frac{-3+2}{2}\right)\\ &= \left(\frac{-2}{2}, \frac{-1}{2}\right)\\ &= \left(-1, -\frac{1}{2}\right)\end{aligned}$$

41. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\\ &= \left(\frac{-0.2+2.3}{2}, \frac{0.3+1.1}{2}\right)\\ &= \left(\frac{2.1}{2}, \frac{1.4}{2}\right)\\ &= (1.05, 0.7)\end{aligned}$$

42. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\\ &= \left(\frac{1.2+(-0.3)}{2}, \frac{2.3+1.1}{2}\right)\\ &= \left(\frac{0.9}{2}, \frac{3.4}{2}\right)\\ &= (0.45, 1.7)\end{aligned}$$

43. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\\ &= \left(\frac{a+0}{2}, \frac{b+0}{2}\right)\\ &= \left(\frac{a}{2}, \frac{b}{2}\right)\end{aligned}$$

44. The coordinates of the midpoint are:

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\\ &= \left(\frac{a+0}{2}, \frac{a+0}{2}\right)\\ &= \left(\frac{a}{2}, \frac{a}{2}\right)\end{aligned}$$

45. Consider points of the form $(2, y)$ that are a distance of 5 units from the point $(-2, -1)$.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\\ &= \sqrt{(-2-2)^2 + (-1-y)^2}\\ &= \sqrt{(-4)^2 + (-1-y)^2}\\ &= \sqrt{16 + 1 + 2y + y^2}\\ &= \sqrt{y^2 + 2y + 17}\end{aligned}$$

$$\begin{aligned}5 &= \sqrt{y^2 + 2y + 17}\\ 5^2 &= \left(\sqrt{y^2 + 2y + 17}\right)^2\\ 25 &= y^2 + 2y + 17\\ 0 &= y^2 + 2y - 8\\ 0 &= (y+4)(y-2)\end{aligned}$$

$$y + 4 = 0 \quad \text{or} \quad y - 2 = 0$$
$$y = -4 \qquad\qquad y = 2$$

Thus, the points $(2, -4)$ and $(2, 2)$ are a distance of 5 units from the point $(-2, -1)$.

46. Consider points of the form $(x, -3)$ that are a distance of 13 units from the point $(1, 2)$.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\\ &= \sqrt{(1-x)^2 + (2-(-3))^2}\\ &= \sqrt{x^2 - 2x + 1 + (5)^2}\\ &= \sqrt{x^2 - 2x + 1 + 25}\\ &= \sqrt{x^2 - 2x + 26}\end{aligned}$$

$$13 = \sqrt{x^2 - 2x + 26}$$
$$13^2 = \left(\sqrt{x^2 - 2x + 26}\right)^2$$
$$169 = x^2 - 2x + 26$$
$$0 = x^2 - 2x - 143$$
$$0 = (x - 13)(x + 11)$$
$$x - 13 = 0 \quad \text{or} \quad x + 11 = 0$$
$$x = 13 \qquad\qquad x = -11$$

Thus, the points $(13, -3)$ and $(-11, -3)$ are a distance of 13 units from the point $(1, 2)$.

47. Points on the x-axis have a y-coordinate of 0. Thus, we consider points of the form $(x, 0)$ that are a distance of 5 units from the point $(4, -3)$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - x)^2 + (-3 - 0)^2} \\ &= \sqrt{16 - 8x + x^2 + (-3)^2} \\ &= \sqrt{16 - 8x + x^2 + 9} \\ &= \sqrt{x^2 - 8x + 25} \end{aligned}$$
$$5 = \sqrt{x^2 - 8x + 25}$$
$$5^2 = \left(\sqrt{x^2 - 8x + 25}\right)^2$$
$$25 = x^2 - 8x + 25$$
$$0 = x^2 - 8x$$
$$0 = x(x - 8)$$
$$x = 0 \quad \text{or} \quad x - 8 = 0$$
$$x = 8$$

Thus, the points $(0, 0)$ and $(8, 0)$ are on the x-axis and a distance of 5 units from the point $(4, -3)$.

48. Points on the y-axis have an x-coordinate of 0. Thus, we consider points of the form $(0, y)$ that are a distance of 5 units from the point $(4, 4)$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - 0)^2 + (4 - y)^2} \\ &= \sqrt{4^2 + 16 - 8y + y^2} \\ &= \sqrt{16 + 16 - 8y + y^2} \\ &= \sqrt{y^2 - 8y + 32} \end{aligned}$$
$$5 = \sqrt{y^2 - 8y + 32}$$
$$5^2 = \left(\sqrt{y^2 - 8y + 32}\right)^2$$
$$25 = y^2 - 8y + 32$$
$$0 = y^2 - 8y + 7$$
$$0 = (y - 7)(y - 1)$$
$$y - 7 = 0 \quad \text{or} \quad y - 1 = 0$$
$$y = 7 \qquad\qquad y = 1$$

Thus, the points $(0, 7)$ and $(0, 1)$ are on the y-axis and a distance of 5 units from the point $(4, 4)$.

49. The midpoint of AB is: $D = \left(\frac{0 + 6}{2}, \frac{0 + 0}{2}\right) = (3, 0)$

The midpoint of AC is: $E = \left(\frac{0 + 4}{2}, \frac{0 + 4}{2}\right) = (2, 2)$

The midpoint of BC is: $F = \left(\frac{6 + 4}{2}, \frac{0 + 4}{2}\right) = (5, 2)$

$$\begin{aligned} d(C, D) &= \sqrt{(0 - 4)^2 + (3 - 4)^2} \\ &= \sqrt{(-4)^2 + (-1)^2} = \sqrt{16 + 1} = \sqrt{17} \end{aligned}$$
$$\begin{aligned} d(B, E) &= \sqrt{(2 - 6)^2 + (2 - 0)^2} \\ &= \sqrt{(-4)^2 + 2^2} = \sqrt{16 + 4} \\ &= \sqrt{20} = 2\sqrt{5} \end{aligned}$$
$$\begin{aligned} d(A, F) &= \sqrt{(2 - 0)^2 + (5 - 0)^2} \\ &= \sqrt{2^2 + 5^2} = \sqrt{4 + 25} \\ &= \sqrt{29} \end{aligned}$$

50. Let $P_1 = (0, 0),\ P_2 = (0, 4),\ P = (x, y)$

$$d(P_1, P_2) = \sqrt{(0-0)^2 + (4-0)^2} = \sqrt{16} = 4$$

$$d(P_1, P) = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2} = 4 \rightarrow x^2 + y^2 = 16$$

$$d(P_2, P) = \sqrt{(x-0)^2 + (y-4)^2} = \sqrt{x^2 + (y-4)^2} = 4 \rightarrow x^2 + (y-4)^2 = 16$$

Therefore,

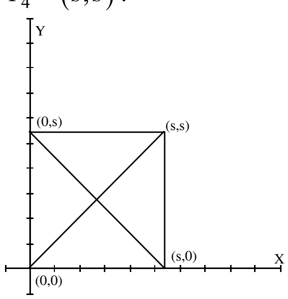
$$y^2 = (y-4)^2$$
$$y^2 = y^2 - 8y + 16$$
$$8y = 16$$
$$y = 2$$

which gives

$$x^2 + 2^2 = 16$$
$$x^2 = 12$$
$$x = \pm 2\sqrt{3}$$

Two triangles are possible. The third vertex is $\left(-2\sqrt{3}, 2\right)$ or $\left(2\sqrt{3}, 2\right)$.

51. Let $P_1 = (0,0)$, $P_2 = (0,s)$, $P_3 = (s,0)$, and $P_4 = (s,s)$.



The points P_1 and P_4 are endpoints of one diagonal and the points P_2 and P_3 are the endpoints of the other diagonal.

$$M_{1,4} = \left(\frac{0+s}{2}, \frac{0+s}{2}\right) = \left(\frac{s}{2}, \frac{s}{2}\right)$$

$$M_{2,3} = \left(\frac{0+s}{2}, \frac{s+0}{2}\right) = \left(\frac{s}{2}, \frac{s}{2}\right)$$

The midpoints of the diagonals are the same. Therefore, the diagonals of a square intersect at their midpoints.

52. Let $P_1 = (0,0)$, $P_2 = (a,0)$, and $P_3 = \left(\frac{a}{2}, \frac{\sqrt{3}\,a}{2}\right)$. To show that these vertices form an equilateral triangle, we need to show that the distance between any pair of points is the same constant value.

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(a-0)^2 + (0-0)^2} = \sqrt{a^2} = |a|$$

$$d(P_2, P_3) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{\left(\frac{a}{2} - a\right)^2 + \left(\frac{\sqrt{3}\,a}{2} - 0\right)^2} = \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}} = \sqrt{\frac{4a^2}{4}} = \sqrt{a^2} = |a|$$

$$d(P_1, P_3) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(\frac{\sqrt{3}\,a}{2} - 0\right)^2} = \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}} = \sqrt{\frac{4a^2}{4}} = \sqrt{a^2} = |a|$$

Since all three distances have the same constant value, the triangle is an equilateral triangle.
Now find the midpoints:

$$D = M_{P_1P_2} = \left(\frac{0+a}{2}, \frac{0+0}{2}\right) = \left(\frac{a}{2}, 0\right)$$

$$E = M_{P_2P_3} = \left(\frac{a + \frac{a}{2}}{2}, \frac{0 + \frac{\sqrt{3}\,a}{2}}{2}\right) = \left(\frac{3a}{4}, \frac{\sqrt{3}\,a}{4}\right)$$

$$F = M_{P_1P_3} = \left(\frac{0 + \frac{a}{2}}{2}, \frac{0 + \frac{\sqrt{3}a}{2}}{2}\right) = \left(\frac{a}{4}, \frac{\sqrt{3}\,a}{4}\right)$$

$$d(D,E)=\sqrt{\left(\frac{3a}{4}-\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}\,a}{4}-0\right)^2}$$
$$=\sqrt{\left(\frac{a}{4}\right)^2+\left(\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$=\sqrt{\frac{a^2}{16}+\frac{3a^2}{16}}=\frac{|a|}{2}$$
$$d(D,F)=\sqrt{\left(\frac{a}{4}-\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}\,a}{4}-0\right)^2}$$
$$=\sqrt{\left(-\frac{a}{4}\right)^2+\left(\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$=\sqrt{\frac{a^2}{16}+\frac{3a^2}{16}}=\frac{|a|}{2}$$
$$d(E,F)=\sqrt{\left(\frac{3a}{4}-\frac{a}{4}\right)^2+\left(\frac{\sqrt{3}\,a}{4}-\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$=\sqrt{\left(\frac{a}{2}\right)^2+0^2}$$
$$=\sqrt{\frac{a^2}{4}}=\frac{|a|}{2}$$

Since the sides are the same length, the triangle is equilateral.

53. $d(P_1,P_2)=\sqrt{(-4-2)^2+(1-1)^2}$
$$=\sqrt{(-6)^2+0^2}$$
$$=\sqrt{36}$$
$$=6$$
$$d(P_2,P_3)=\sqrt{(-4-(-4))^2+(-3-1)^2}$$
$$=\sqrt{0^2+(-4)^2}$$
$$=\sqrt{16}$$
$$=4$$
$$d(P_1,P_3)=\sqrt{(-4-2)^2+(-3-1)^2}$$
$$=\sqrt{(-6)^2+(-4)^2}$$
$$=\sqrt{36+16}$$
$$=\sqrt{52}$$
$$=2\sqrt{13}$$

Since $\left[d(P_1,P_2)\right]^2+\left[d(P_2,P_3)\right]^2=\left[d(P_1,P_3)\right]^2$, the triangle is a right triangle.

54. $d(P_1,P_2)=\sqrt{(6-(-1))^2+(2-4)^2}$
$$=\sqrt{7^2+(-2)^2}$$
$$=\sqrt{49+4}$$
$$=\sqrt{53}$$
$$d(P_2,P_3)=\sqrt{(4-6)^2+(-5-2)^2}$$
$$=\sqrt{(-2)^2+(-7)^2}$$
$$=\sqrt{4+49}$$
$$=\sqrt{53}$$
$$d(P_1,P_3)=\sqrt{(4-(-1))^2+(-5-4)^2}$$
$$=\sqrt{5^2+(-9)^2}$$
$$=\sqrt{25+81}$$
$$=\sqrt{106}$$

Since $\left[d(P_1,P_2)\right]^2+\left[d(P_2,P_3)\right]^2=\left[d(P_1,P_3)\right]^2$, the triangle is a right triangle.

Since $d(P_1,P_2)=d(P_2,P_3)$, the triangle is isosceles.

Therefore, the triangle is an isosceles right triangle.

55. $d(P_1,P_2)=\sqrt{(0-(-2))^2+(7-(-1))^2}$
$$=\sqrt{2^2+8^2}=\sqrt{4+64}=\sqrt{68}$$
$$=2\sqrt{17}$$
$$d(P_2,P_3)=\sqrt{(3-0)^2+(2-7)^2}$$
$$=\sqrt{3^2+(-5)^2}=\sqrt{9+25}$$
$$=\sqrt{34}$$
$$d(P_1,P_3)=\sqrt{(3-(-2))^2+(2-(-1))^2}$$
$$=\sqrt{5^2+3^2}=\sqrt{25+9}$$
$$=\sqrt{34}$$

Since $d(P_2,P_3)=d(P_1,P_3)$, the triangle is isosceles.

Since $\left[d(P_1,P_3)\right]^2+\left[d(P_2,P_3)\right]^2=\left[d(P_1,P_2)\right]^2$, the triangle is also a right triangle.

Therefore, the triangle is an isosceles right triangle.

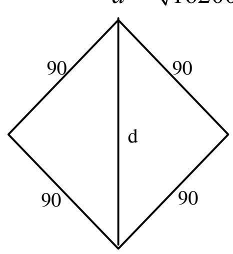
56. $d(P_1,P_2)=\sqrt{\left(-4-7\right)^2+\left(0-2\right)^2}$
$=\sqrt{(-11)^2+(-2)^2}$
$=\sqrt{121+4}=\sqrt{125}$
$=5\sqrt{5}$

$d(P_2,P_3)=\sqrt{\left(4-(-4)\right)^2+(6-0)^2}$
$=\sqrt{8^2+6^2}=\sqrt{64+36}$
$=\sqrt{100}$
$=10$

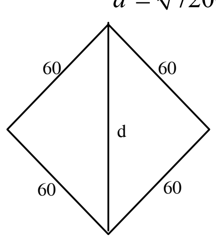
$d(P_1,P_3)=\sqrt{\left(4-7\right)^2+\left(6-2\right)^2}$
$=\sqrt{(-3)^2+4^2}=\sqrt{9+16}$
$=\sqrt{25}$
$=5$

Since $\left[d(P_1,P_3)\right]^2+\left[d(P_2,P_3)\right]^2=\left[d(P_1,P_2)\right]^2$, the triangle is a right triangle.

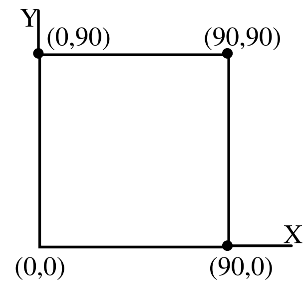
57. Using the Pythagorean Theorem:
$90^2+90^2=d^2$
$8100+8100=d^2$
$16200=d^2$
$d=\sqrt{16200}=90\sqrt{2}\approx 127.28$ feet



58. Using the Pythagorean Theorem:
$60^2+60^2=d^2$
$3600+3600=d^2 \rightarrow 7200=d^2$
$d=\sqrt{7200}=60\sqrt{2}\approx 84.85$ feet



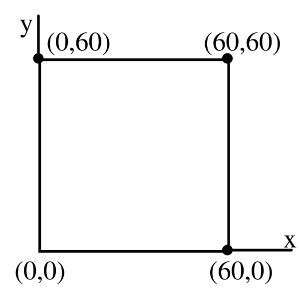
59. a. First: (90, 0), Second: (90, 90)
Third: (0, 90)



b. Using the distance formula:
$d=\sqrt{(310-90)^2+(15-90)^2}$
$=\sqrt{220^2+(-75)^2}=\sqrt{54025}$
$=5\sqrt{2161}\approx 232.43$ feet

c. Using the distance formula:
$d=\sqrt{(300-0)^2+(300-90)^2}$
$=\sqrt{300^2+210^2}=\sqrt{134100}$
$=30\sqrt{149}\approx 366.20$ feet

60. a. First: (60, 0), Second: (60, 60)
Third: (0, 60)

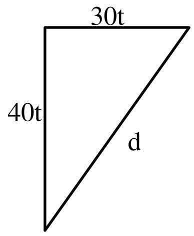


b. Using the distance formula:
$d=\sqrt{(180-60)^2+(20-60)^2}$
$=\sqrt{120^2+(-40)^2}=\sqrt{16000}$
$=40\sqrt{10}\approx 126.49$ feet

c. Using the distance formula:
$d=\sqrt{(220-0)^2+(220-60)^2}$
$=\sqrt{220^2+160^2}=\sqrt{74000}$
$=20\sqrt{185}\approx 272.03$ feet

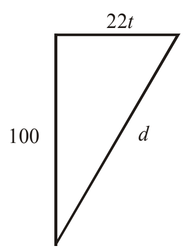
61. The Neon heading east moves a distance $30t$ after t hours. The truck heading south moves a distance $40t$ after t hours. Their distance apart after t hours is:

$$\begin{aligned} d &= \sqrt{(30t)^2 + (40t)^2} \\ &= \sqrt{900t^2 + 1600t^2} \\ &= \sqrt{2500t^2} \\ &= 50t \text{ miles} \end{aligned}$$



62. $\dfrac{15 \text{ miles}}{1 \text{ hr}} \cdot \dfrac{5280 \text{ ft}}{1 \text{ mile}} \cdot \dfrac{1 \text{ hr}}{3600 \text{ sec}} = 22 \text{ ft/sec}$

$$\begin{aligned} d &= \sqrt{100^2 + (22t)^2} \\ &= \sqrt{10000 + 484t^2} \text{ feet} \end{aligned}$$



Section F.2

1.
$$\begin{aligned} 2(x+3) - 1 &= -7 \\ 2(x+3) &= -6 \\ x + 3 &= -3 \\ x &= -6 \end{aligned}$$
The solution set is $\{-6\}$.

2.
$$\begin{aligned} x^2 - 9 &= 0 \\ x^2 &= 9 \\ x &= \pm\sqrt{9} \\ x &= \pm 3 \end{aligned}$$
The solution set is $\{-3, 3\}$.

3. intercepts

4. zeros; roots

5. y-axis

6. 4

7. $(-3, 4)$

8. True

9. False; the y-coordinate of a point at which the graph crosses or touches the x-axis is always 0. The x-coordinate of such a point is an x-intercept.

10. False; a graph can be symmetric with respect to both coordinate axes (in such cases it will also be symmetric with respect to the origin). For example: $x^2 + y^2 = 1$

11. $y = x^4 - \sqrt{x}$

$0 = 0^4 - \sqrt{0}$ $1 = 1^4 - \sqrt{1}$ $0 = (-1)^4 - \sqrt{-1}$
$0 = 0$ $1 \neq 0$ $0 \neq 1 - \sqrt{-1}$

(0, 0) is on the graph of the equation.

12. $y = x^3 - 2\sqrt{x}$

$0 = 0^3 - 2\sqrt{0}$ $1 = 1^3 - 2\sqrt{1}$ $-1 = 1^3 - 2\sqrt{1}$
$0 = 0$ $1 \neq -1$ $-1 = -1$

(0, 0) and (1, –1) are on the graph of the equation.

13. $y^2 = x^2 + 9$

$3^2 = 0^2 + 9$ $0^2 = 3^2 + 9$ $0^2 = (-3)^2 + 9$
$9 = 9$ $0 \neq 18$ $0 \neq 18$

(0, 3) is on the graph of the equation.

14. $y^3 = x + 1$

$2^3 = 1 + 1$ $1^3 = 0 + 1$ $0^3 = -1 + 1$
$8 \neq 2$ $1 = 1$ $0 = 0$

(0, 1) and (–1, 0) are on the graph of the equation.

15. $x^2 + y^2 = 4$

$0^2 + 2^2 = 4$ $(-2)^2 + 2^2 = 4$ $\sqrt{2}^2 + \sqrt{2}^2 = 4$
$4 = 4$ $8 \neq 4$ $4 = 4$

(0, 2) and $\left(\sqrt{2}, \sqrt{2}\right)$ are on the graph of the equation.

16. $x^2 + 4y^2 = 4$

$0^2 + 4 \cdot 1^2 = 4 \qquad 2^2 + 4 \cdot 0^2 = 4 \qquad 2^2 + 4\left(\frac{1}{2}\right)^2 = 4$

$\qquad 4 = 4 \qquad\qquad 4 = 4 \qquad\qquad 5 \neq 4$

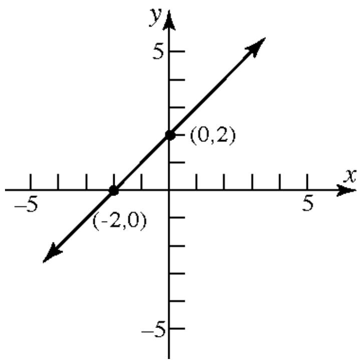
(0, 1) and (2, 0) are on the graph of the equation.

17. $y = x + 2$

x-intercept: $0 = x + 2$; $-2 = x$

y-intercept: $y = 0 + 2$; $y = 2$

The intercepts are $(-2,0)$ and $(0,2)$.

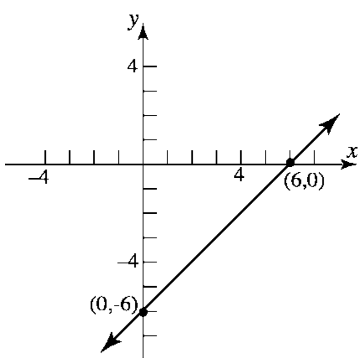


18. $y = x - 6$

x-intercept: $0 = x - 6$; $6 = x$

y-intercept: $y = 0 - 6$; $y = -6$

The intercepts are $(6,0)$ and $(0,-6)$.

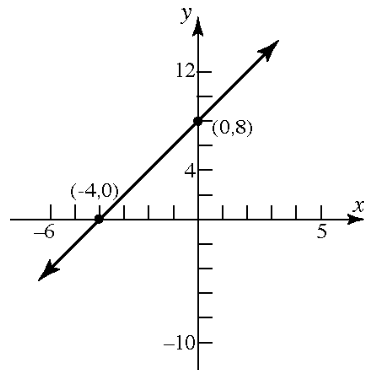


19. $y = 2x + 8$

x-intercept: $0 = 2x + 8$; $2x = -8$; $x = -4$

y-intercept: $y = 2(0) + 8$; $y = 8$

The intercepts are $(-4,0)$ and $(0,8)$.

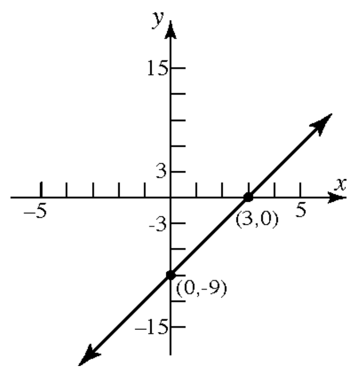


20. $y = 3x - 9$

x-intercept: $0 = 3x - 9$; $3x = 9$; $x = 3$

y-intercept: $y = 3(0) - 9$; $y = -9$

The intercepts are $(3,0)$ and $(0,-9)$.

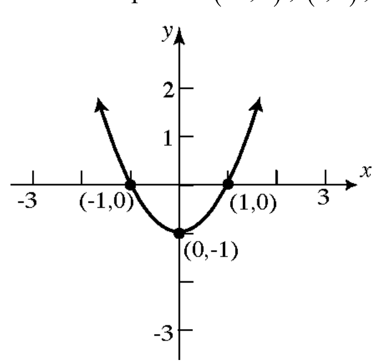


21. $y = x^2 - 1$

x-intercepts: $0 = x^2 - 1$; $x^2 = 1$; $x = \pm 1$

y-intercept: $y = 0^2 - 1$; $y = -1$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,-1)$.

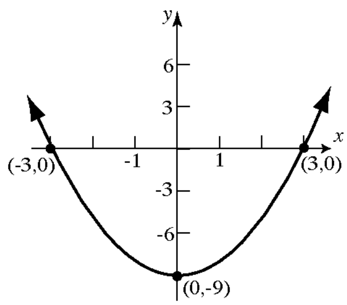


22. $y = x^2 - 9$

x-intercepts:
$0 = x^2 - 9$
$x^2 = 9$
$x = \pm 3$

y-intercept:
$y = 0^2 - 9$
$y = -9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,-9)$.

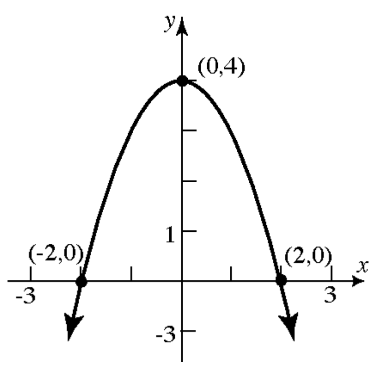


23. $y = -x^2 + 4$

x-intercepts:
$0 = -x^2 + 4$
$x^2 = 4$
$x = \pm 2$

y-intercepts:
$y = -(0)^2 + 4$
$y = 4$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,4)$.

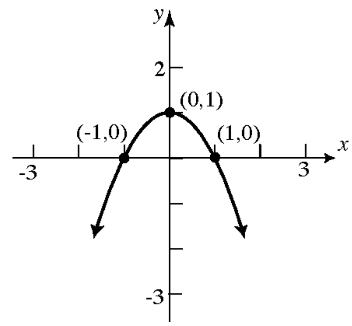


24. $y = -x^2 + 1$

x-intercepts:
$0 = -x^2 + 1$
$x^2 = 1$
$x = \pm 1$

y-intercept:
$y = -(0)^2 + 1$
$y = 1$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,1)$.

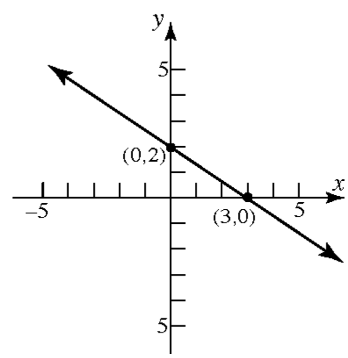


25. $2x + 3y = 6$

x-intercepts:
$2x + 3(0) = 6$
$2x = 6$
$x = 3$

y-intercept:
$2(0) + 3y = 6$
$3y = 6$
$y = 2$

The intercepts are $(3,0)$ and $(0,2)$.

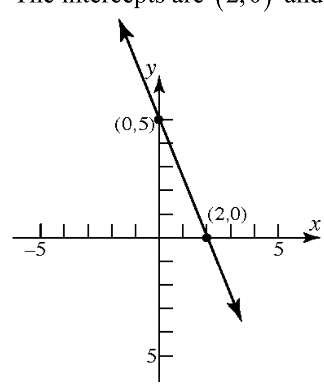


26. $5x + 2y = 10$

x-intercepts:
$5x + 2(0) = 10$
$5x = 10$
$x = 2$

y-intercept:
$5(0) + 2y = 10$
$2y = 10$
$y = 5$

The intercepts are $(2,0)$ and $(0,5)$.

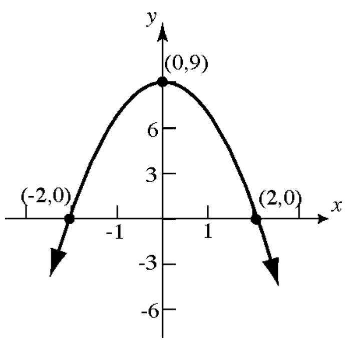


27. $9x^2 + 4y = 36$

x-intercepts:
$$9x^2 + 4(0) = 36$$
$$9x^2 = 36$$
$$x^2 = 4$$
$$x = \pm 2$$

y-intercept:
$$9(0)^2 + 4y = 36$$
$$4y = 36$$
$$y = 9$$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,9)$.

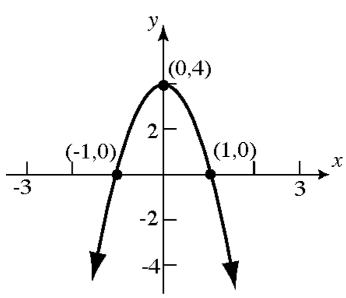


28. $4x^2 + y = 4$

x-intercepts:
$$4x^2 + 0 = 4$$
$$4x^2 = 4$$
$$x^2 = 1$$
$$x = \pm 1$$

y-intercept:
$$4(0)^2 + y = 4$$
$$y = 4$$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,4)$.



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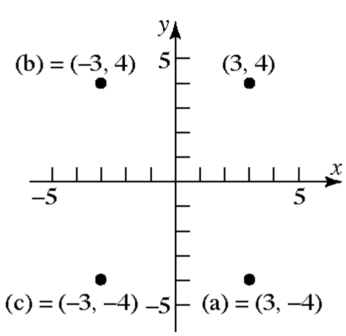

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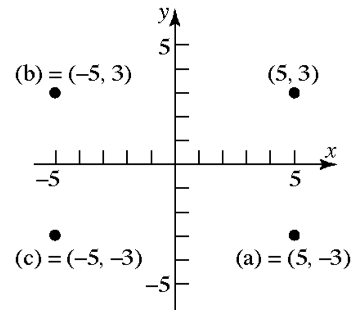

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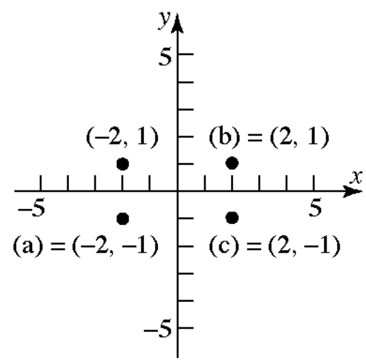

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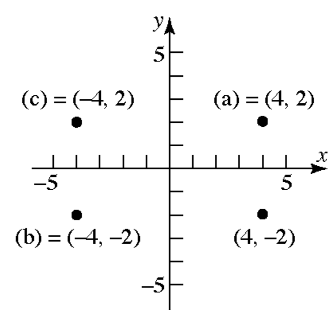

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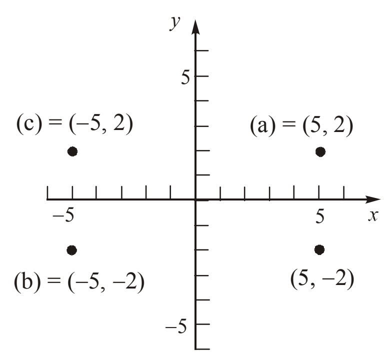

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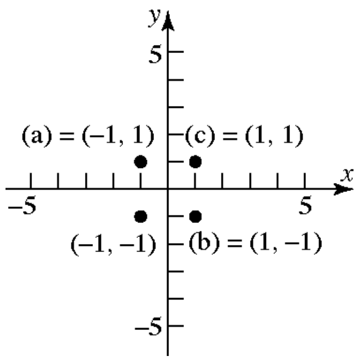

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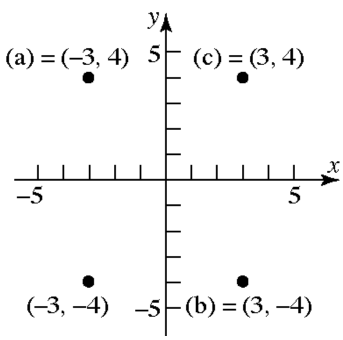

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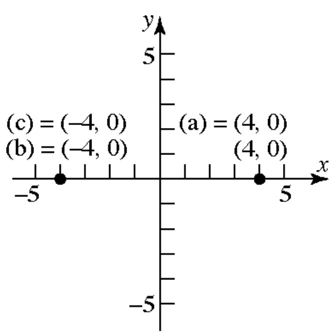

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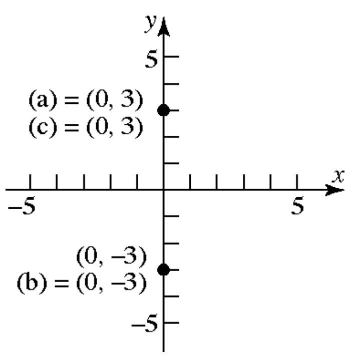

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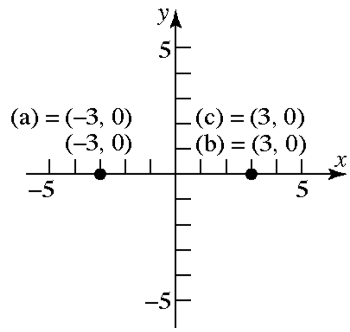


39. **a.** Intercepts: $(-1,0)$ and $(1,0)$

b. Symmetric with respect to the x-axis, y-axis, and the origin.

40. **a.** Intercepts: $(0,1)$

b. Not symmetric to x-axis, y-axis, or origin

41. **a.** Intercepts: $\left(-\frac{\pi}{2},0\right)$, $(0,1)$, and $\left(\frac{\pi}{2},0\right)$

b. Symmetric with respect to the y-axis.

42. **a.** Intercepts: $(-2,0)$, $(0,-3)$, and $(2,0)$

b. Symmetric with respect to the y-axis.

43. **a.** Intercepts: $(0,0)$

b. Symmetric with respect to the x-axis.

44. **a.** Intercepts: $(-2,0)$, $(0,2)$, $(0,-2)$, and $(2,0)$

b. Symmetric with respect to the x-axis, y-axis, and the origin.

45. **a.** Intercepts: $(-2,0)$, $(0,0)$, and $(2,0)$

b. Symmetric with respect to the origin.

46. **a.** Intercepts: $(-4,0)$, $(0,0)$, and $(4,0)$

b. Symmetric with respect to the origin.

47. **a.** Intercepts: $(-1,0)$, $(0,-1)$, $(1,0)$

b. Symmetric with respect to the *y*-axis.

48. a. Intercepts: $(0,0)$

b. Symmetric with respect to the origin.

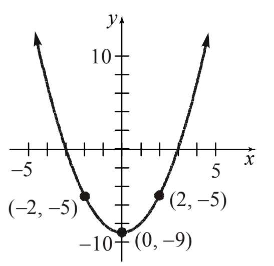
49. a. Intercepts: none

b. Symmetric with respect to the origin.

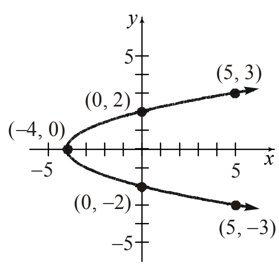
50. a. Intercepts: none

b. Symmetric with respect to the x-axis.

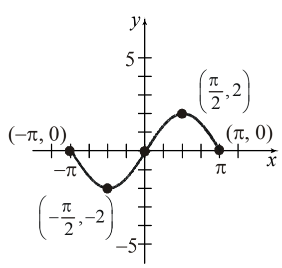
51.



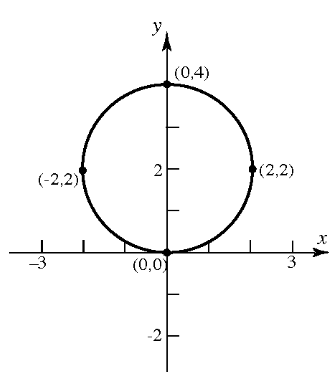
52.



53.



54.



55. $y^2 = x + 4$

x-intercepts:
$0^2 = x + 4$
$-4 = x$

y-intercepts:
$y^2 = 0 + 4$
$y^2 = 4$
$y = \pm 2$

The intercepts are $(-4,0)$, $(0,-2)$ and $(0,2)$.

Test x-axis symmetry: Let $y = -y$

$(-y)^2 = x + 4$

$y^2 = x + 4$ same

Test y-axis symmetry: Let $x = -x$

$y^2 = -x + 4$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.

$(-y)^2 = -x + 4$

$y^2 = -x + 4$ different

Therefore, the graph will have x-axis symmetry.

56. $y^2 = x + 9$

x-intercepts:
$0^2 = x + 9$
$-9 = x$

y-intercepts:
$y^2 = 0 + 9$
$y^2 = 9$
$y = \pm 3$

The intercepts are $(-9,0)$, $(0,-3)$ and $(0,3)$.

Test x-axis symmetry: Let $y = -y$

$(-y)^2 = x + 9$

$y^2 = x + 9$ same

Test y-axis symmetry: Let $x = -x$

$y^2 = -x + 9$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.

$(-y)^2 = -x + 9$

$y^2 = -x + 9$ different

Therefore, the graph will have x-axis symmetry.

57. $y = \sqrt[3]{x}$

x-intercepts: $0 = \sqrt[3]{x}$, $0 = x$

y-intercepts: $y = \sqrt[3]{0} = 0$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \sqrt[3]{x}$ different

Test y-axis symmetry: Let $x = -x$

$y = \sqrt[3]{-x} = -\sqrt[3]{x}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \sqrt[3]{-x} = -\sqrt[3]{x}$

$y = \sqrt[3]{x}$ same

Therefore, the graph will have origin symmetry.

58. $y = \sqrt[5]{x}$

x-intercepts: $0 = \sqrt[3]{x}$, $0 = x$

y-intercepts: $y = \sqrt[5]{0} = 0$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \sqrt[5]{x}$ different

Test y-axis symmetry: Let $x = -x$

$y = \sqrt[5]{-x} = -\sqrt[5]{x}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \sqrt[5]{-x} = -\sqrt[5]{x}$

$y = \sqrt[5]{x}$ same

Therefore, the graph will have origin symmetry.

59. $x^2 + y - 9 = 0$

x-intercepts: $x^2 - 9 = 0$, $x^2 = 9$, $x = \pm 3$

y-intercepts: $0^2 + y - 9 = 0$, $y = 9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,9)$.

Test x-axis symmetry: Let $y = -y$

$x^2 - y - 9 = 0$ different

Test y-axis symmetry: Let $x = -x$

$(-x)^2 + y - 9 = 0$

$x^2 + y - 9 = 0$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$(-x)^2 - y - 9 = 0$

$x^2 - y - 9 = 0$ different

Therefore, the graph will have y-axis symmetry.

60. $x^2 - y - 4 = 0$

x-intercepts: $x^2 - 0 - 4 = 0$, $x^2 = 4$, $x = \pm 2$

y-intercept: $0^2 - y - 4 = 0$, $-y = 4$, $y = -4$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,-4)$.

Test x-axis symmetry: Let $y = -y$

$x^2 - (-y) - 4 = 0$

$x^2 + y - 4 = 0$ different

Test y-axis symmetry: Let $x = -x$

$(-x)^2 - y - 4 = 0$

$x^2 - y - 4 = 0$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$(-x)^2 - (-y) - 4 = 0$

$x^2 + y - 4 = 0$ different

Therefore, the graph will have y-axis symmetry.

61. $9x^2 + 4y^2 = 36$

x-intercepts: $9x^2 + 4(0)^2 = 36$, $9x^2 = 36$, $x^2 = 4$, $x = \pm 2$

y-intercepts: $9(0)^2 + 4y^2 = 36$, $4y^2 = 36$, $y^2 = 9$, $y = \pm 3$

The intercepts are $(-2,0)$, $(2,0)$, $(0,-3)$, and $(0,3)$.

Test x-axis symmetry: Let $y = -y$

$9x^2 + 4(-y)^2 = 36$

$9x^2 + 4y^2 = 36$ same

Test y-axis symmetry: Let $x = -x$

$9(-x)^2 + 4y^2 = 36$

$9x^2 + 4y^2 = 36$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$9(-x)^2 + 4(-y)^2 = 36$

$9x^2 + 4y^2 = 36$ same

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

62. $4x^2 + y^2 = 4$

x-intercepts:

$4x^2 + 0^2 = 4$

$4x^2 = 4$

$x^2 = 1$

$x = \pm 1$

y-intercepts:

$4(0)^2 + y^2 = 4$

$y^2 = 4$

$y = \pm 2$

The intercepts are $(-1,0)$, $(1,0)$, $(0,-2)$, and $(0,2)$.

Test x-axis symmetry: Let $y = -y$

$4x^2 + (-y)^2 = 4$

$4x^2 + y^2 = 4$ same

Test y-axis symmetry: Let $x = -x$

$4(-x)^2 + y^2 = 4$

$4x^2 + y^2 = 4$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$4(-x)^2 + (-y)^2 = 4$

$4x^2 + y^2 = 4$ same

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

63. $y = x^3 - 27$

x-intercepts:

$0 = x^3 - 27$

$x^3 = 27$

$x = 3$

y-intercepts:

$y = 0^3 - 27$

$y = -27$

The intercepts are $(3,0)$ and $(0,-27)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^3 - 27$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^3 - 27$

$y = -x^3 - 27$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^3 - 27$

$y = x^3 + 27$ different

Therefore, the graph has none of the indicated symmetries.

64. $y = x^4 - 1$

x-intercepts:

$0 = x^4 - 1$

$x^4 = 1$

$x = \pm 1$

y-intercepts:

$y = 0^4 - 1$

$y = -1$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,-1)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^4 - 1$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^4 - 1$

$y = x^4 - 1$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^4 - 1$

$-y = x^4 - 1$ different

Therefore, the graph will have y-axis symmetry.

65. $y = x^2 - 3x - 4$

x-intercepts:

$0 = x^2 - 3x - 4$

$0 = (x-4)(x+1)$

$x = 4$ or $x = -1$

y-intercepts:

$y = 0^2 - 3(0) - 4$

$y = -4$

The intercepts are $(4,0)$, $(-1,0)$, and $(0,-4)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^2 - 3x - 4$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^2 - 3(-x) - 4$

$y = x^2 + 3x - 4$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^2 - 3(-x) - 4$

$-y = x^2 + 3x - 4$ different

Therefore, the graph has none of the indicated symmetries.

66. $y = x^2 + 4$

x-intercepts:

$0 = x^2 + 4$

$x^2 = -4$

no real solution

y-intercepts:

$y = 0^2 + 4$

$y = 4$

The only intercept is $(0, 4)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^2 + 4$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^2 + 4$

$y = x^2 + 4$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^2 + 4$

$-y = x^2 + 4$ different

Therefore, the graph will have y-axis symmetry.

67. $y = \dfrac{3x}{x^2 + 9}$

x-intercepts:

$0 = \dfrac{3x}{x^2 + 9}$

$3x = 0$

$x = 0$

y-intercepts:

$y = \dfrac{3(0)}{0^2 + 9} = \dfrac{0}{9} = 0$

The only intercept is $(0, 0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \dfrac{3x}{x^2 + 9}$ different

Test y-axis symmetry: Let $x = -x$

$y = \dfrac{3(-x)}{(-x)^2 + 9}$

$y = -\dfrac{3x}{x^2 + 9}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \dfrac{3(-x)}{(-x)^2 + 9}$

$-y = -\dfrac{3x}{x^2 + 9}$

$y = \dfrac{3x}{x^2 + 9}$ same

Therefore, the graph has origin symmetry.

68. $y = \dfrac{x^2 - 4}{2x}$

x-intercepts:

$0 = \dfrac{x^2 - 4}{2x}$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

y-intercepts:

$y = \dfrac{0^2 - 4}{2(0)} = \dfrac{-4}{0}$

undefined

The intercepts are $(-2, 0)$ and $(2, 0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \dfrac{x^2 - 4}{2x}$ different

Test y-axis symmetry: Let $x = -x$

$y = \dfrac{(-x)^2 - 4}{2(-x)}$

$y = -\dfrac{x^2 - 4}{2x}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \dfrac{(-x)^2 - 4}{2(-x)}$

$y = \dfrac{x^2 - 4}{2x}$ same

Therefore, the graph has origin symmetry.

69. $y = \dfrac{-x^3}{x^2 - 9}$

x-intercepts:

$0 = \dfrac{-x^3}{x^2 - 9}$

$-x^3 = 0$

$x = 0$

y-intercepts:

$y = \dfrac{-0^3}{0^2 - 9} = \dfrac{0}{-9} = 0$

The only intercept is $(0, 0)$.

Test x-axis symmetry: Let $y = -y$

$$-y = \frac{-x^3}{x^2 - 9}$$

$$y = \frac{x^3}{x^2 - 9} \quad \text{different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \frac{-(-x)^3}{(-x)^2 - 9}$$

$$y = \frac{x^3}{x^2 - 9} \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \frac{-(-x)^3}{(-x)^2 - 9}$$

$$-y = \frac{x^3}{x^2 - 9}$$

$$y = \frac{-x^3}{x^2 - 9} \quad \text{same}$$

Therefore, the graph has origin symmetry.

70. $y = \dfrac{x^4 + 1}{2x^5}$

x-intercepts:

$$0 = \frac{x^4 + 1}{2x^5}$$

$$x^4 = -1$$

no real solution

y-intercepts:

$$y = \frac{0^4 + 1}{2(0)^5} = \frac{1}{0}$$

undefined

There are no intercepts for the graph of this equation.

Test x-axis symmetry: Let $y = -y$

$$-y = \frac{x^4 + 1}{2x^5} \quad \text{different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \frac{(-x)^4 + 1}{2(-x)^5}$$

$$y = \frac{x^4 + 1}{-2x^5} \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

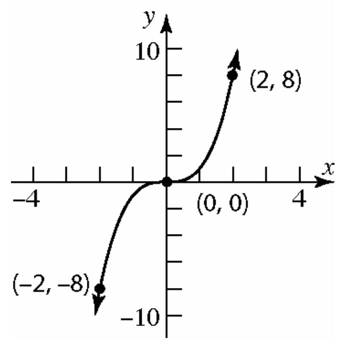
$$-y = \frac{(-x)^4 + 1}{2(-x)^5}$$

$$-y = \frac{x^4 + 1}{-2x^5}$$

$$y = \frac{x^4 + 1}{2x^5} \quad \text{same}$$

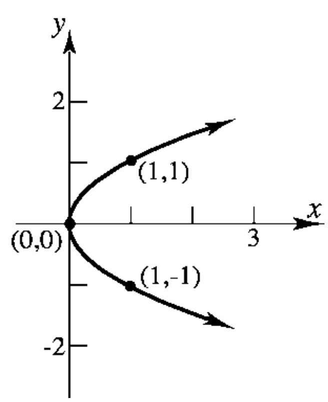
Therefore, the graph has origin symmetry.

71. $y = x^3$



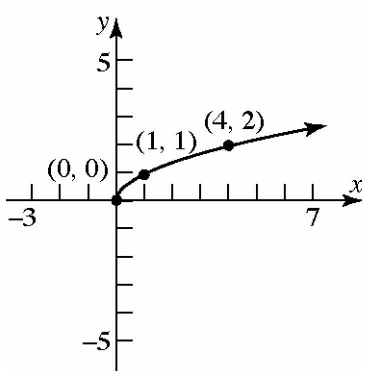
Intercept: $(0,0)$

72. $x = y^2$



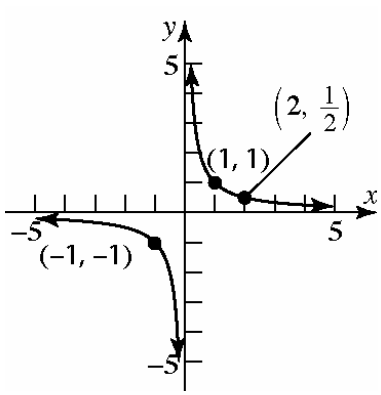
Intercept: $(0,0)$

73. $y = \sqrt{x}$



Intercept: $(0,0)$

74. $y = \dfrac{1}{x}$



no x or y intercepts

75. If the point $(3,b)$ is on the graph of $y = 4x+1$, then we have

$b = 4(3)+1 = 12+1 = 13$

Thus, $b = 13$.

76. If the point $(-2,b)$ is on the graph of $2x+3y=2$, then we have

$$2(-2)+3(b) = 2$$
$$-4+3b = 2$$
$$3b = 6$$
$$b = 2$$

Thus, $b = 2$.

77. If the point $(a,4)$ is on the graph of $y = x^2+3x$, then we have

$$4 = a^2+3a$$
$$0 = a^2+3a-4$$
$$0 = (a+4)(a-1)$$
$$a+4 = 0 \quad \text{or} \quad a-1=0$$
$$a = -4 \qquad\qquad a = 1$$

Thus, $a = -4$ or $a = 1$.

78. If the point $(a,-5)$ is on the graph of $y = x^2+6x$, then we have

$$-5 = a^2+6a$$
$$0 = a^2+6a+5$$
$$0 = (a+5)(a+1)$$
$$a+5 = 0 \quad \text{or} \quad a+1=0$$
$$a = -5 \qquad\qquad a = -1$$

Thus, $a = -5$ or $a = -1$.

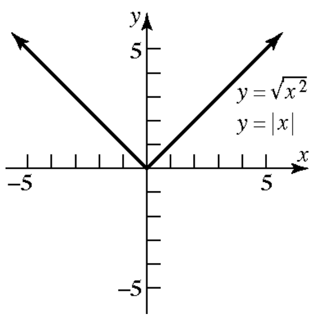
79. For a graph with origin symmetry, if the point (a,b) is on the graph, then so is the point $(-a,-b)$. Since the point $(1,2)$ is on the graph of an equation with origin symmetry, the point $(-1,-2)$ must also be on the graph.

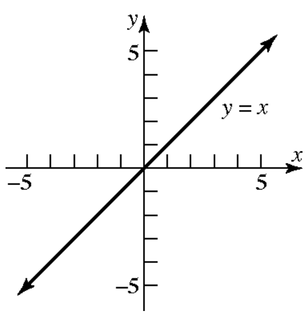
80. For a graph with y-axis symmetry, if the point (a,b) is on the graph, then so is the point $(-a,b)$. Since 6 is an x-intercept in this case, the point $(6,0)$ is on the graph of the equation. Due to the y-axis symmetry, the point $(-6,0)$ must also be on the graph. Therefore, -6 is another x-intercept.

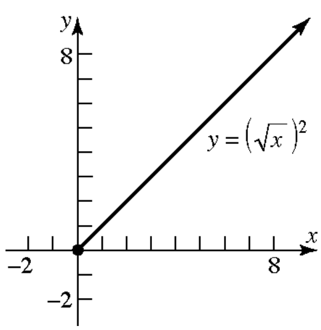
81. For a graph with origin symmetry, if the point (a,b) is on the graph, then so is the point $(-a,-b)$. Since -4 is an x-intercept in this case, the point $(-4,0)$ is on the graph of the equation. Due to the origin symmetry, the point $(4,0)$ must also be on the graph. Therefore, 4 is another x-intercept.

82. For a graph with x-axis symmetry, if the point (a,b) is on the graph, then so is the point $(a,-b)$. Since 2 is a y-intercept in this case, the point $(0,2)$ is on the graph of the equation. Due to the x-axis symmetry, the point $(0,-2)$ must also be on the graph. Therefore, -2 is another y-intercept.

83. **a.**







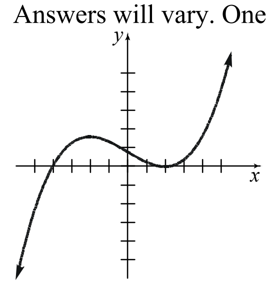
b. Since $\sqrt{x^2}=|x|$ for all x, the graphs of $y=\sqrt{x^2}$ and $y=|x|$ are the same.

c. For $y=\left(\sqrt{x}\right)^2$, the domain of the variable x is $x\ge 0$; for $y=x$, the domain of the variable x is all real numbers. Thus, $\left(\sqrt{x}\right)^2=x$ only for $x\ge 0$.

d. For $y=\sqrt{x^2}$, the range of the variable y is $y\ge 0$; for $y=x$, the range of the variable y is all real numbers. Also, $\sqrt{x^2}=x$ only if $x\ge 0$. Otherwise, $\sqrt{x^2}=-x$.

84. Answers will vary. A complete graph presents enough of the graph to the viewer so they can "see" the rest of the graph as an obvious continuation of what is shown.

85. Answers will vary. One example:



86. Answers will vary

87. Answers will vary

88. Answers will vary.
Case 1: Graph has x-axis and y-axis symmetry, show origin symmetry.
(x,y) on graph $\to (x,-y)$ on graph
(from x-axis symmetry)
$(x,-y)$ on graph $\to (-x,-y)$ on graph
(from y-axis symmetry)
Since the point $(-x,-y)$ is also on the graph, the graph has origin symmetry.

Case 2: Graph has x-axis and origin symmetry, show y-axis symmetry.
(x,y) on graph $\to (x,-y)$ on graph
(from x-axis symmetry)
$(x,-y)$ on graph $\to (-x,y)$ on graph
(from origin symmetry)
Since the point $(-x,y)$ is also on the graph, the graph has y-axis symmetry.

Case 3: Graph has y-axis and origin symmetry, show x-axis symmetry.

(x, y) on graph $\to (-x, y)$ on graph

(from y-axis symmetry)

$(-x, y)$ on graph $\to (x, -y)$ on graph

(from origin symmetry)

Since the point $(x, -y)$ is also on the graph, the graph has x-axis symmetry.

Section F.3

1. undefined; 0

2. 3; 2

x-intercept: $2x + 3(0) = 6$

$$2x = 6$$
$$x = 3$$

y-intercept: $2(0) + 3y = 6$

$$3y = 6$$
$$y = 2$$

3. $y = b$; y-intercept

4. True

5. False; the slope is $\frac{3}{2}$.

$$2y = 3x + 5$$
$$y = \frac{3}{2}x + \frac{5}{2}$$

6. True; $2(1) + (2) \stackrel{?}{=} 4$

$$2 + 2 \stackrel{?}{=} 4$$
$$4 = 4 \checkmark$$

7. $m_1 = m_2$; y-intercepts; $m_1 \cdot m_2 = -1$

8. 2

9. $-\frac{1}{2}$

10. False; perpendicular lines have slopes that are opposite-reciprocals of each other.

11. a. Slope $= \frac{1-0}{2-0} = \frac{1}{2}$

b. If x increases by 2 units, y will increase by 1 unit.

12. a. Slope $= \frac{1-0}{-2-0} = -\frac{1}{2}$

b. If x increases by 2 units, y will decrease by 1 unit.

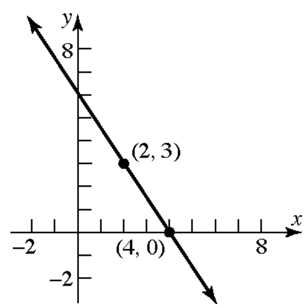
13. a. Slope $= \frac{1-2}{1-(-2)} = -\frac{1}{3}$

b. If x increases by 3 units, y will decrease by 1 unit.

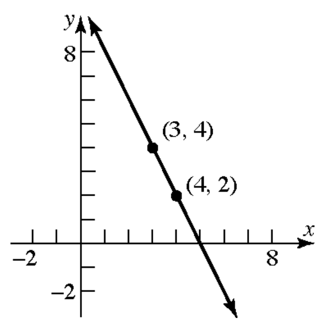
14. a. Slope $= \frac{2-1}{2-(-1)} = \frac{1}{3}$

b. If x increases by 3 units, y will increase by 1 unit.

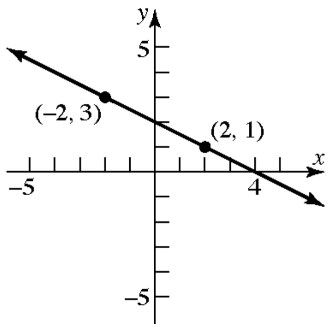
15. Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{4-2} = -\frac{3}{2}$



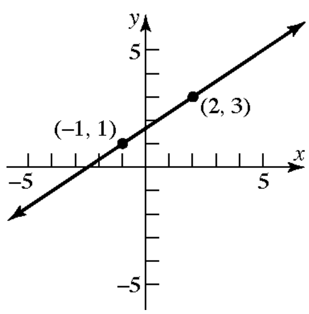
16. Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{3-4} = \frac{2}{-1} = -2$



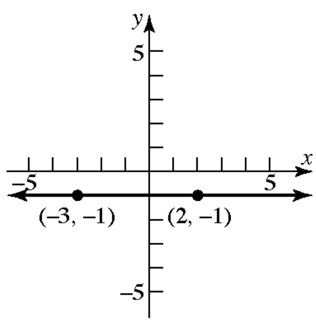
17. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-3}{2-(-2)} = \dfrac{-2}{4} = -\dfrac{1}{2}$



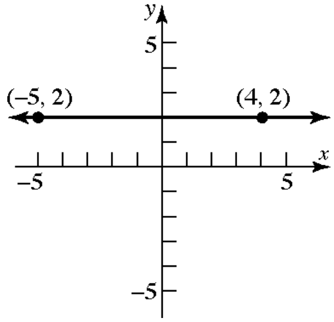
18. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-1}{2-(-1)} = \dfrac{2}{3}$



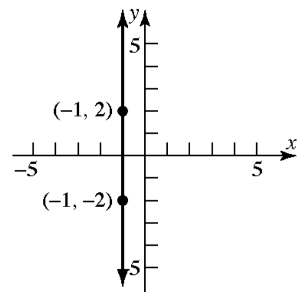
19. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-(-1)}{2-(-3)} = \dfrac{0}{5} = 0$



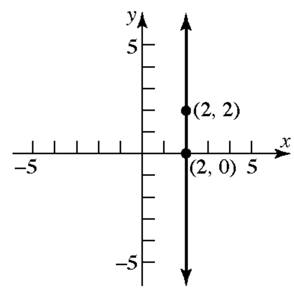
20. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-2}{-5-4} = \dfrac{0}{-9} = 0$



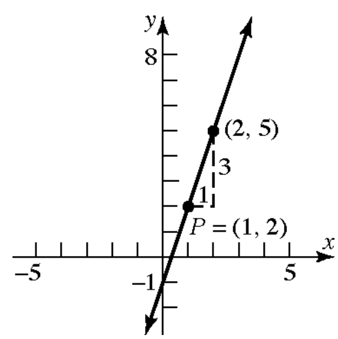
21. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-2}{-1-(-1)} = \dfrac{-4}{0}$ undefined.



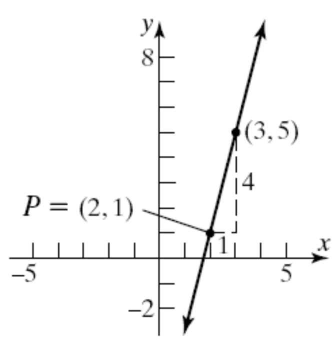
22. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-0}{2-2} = \dfrac{2}{0}$ undefined.



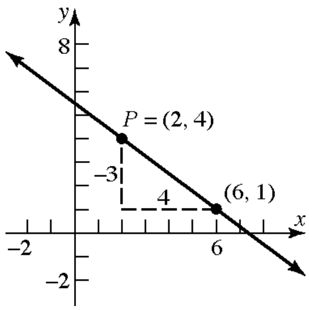
23. $P = (1,2); m = 3$



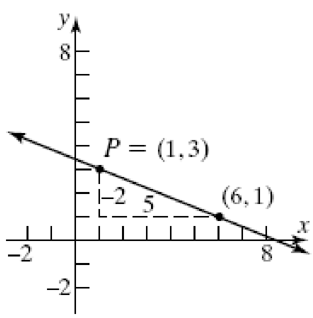
24. $P = (2,1); m = 4$



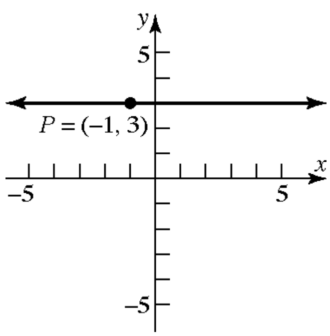
25. $P=(2,4); m=-\frac{3}{4}$



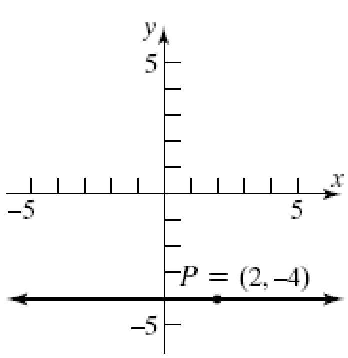
26. $P=(1,3); m=-\frac{2}{5}$



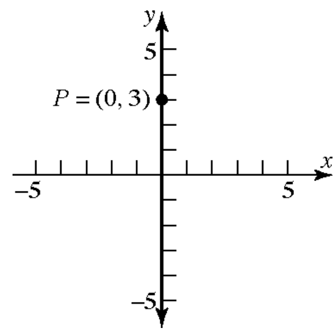
27. $P=(-1,3); m=0$



28. $P=(2,-4); m=0$

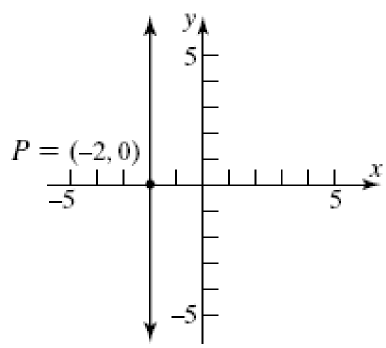


29. $P=(0,3)$; slope undefined



(note: the line is the y-axis)

30. $P=(-2,0)$; slope undefined



31. Slope $= 4 = \frac{4}{1}$; point: $(1,2)$

If x increases by 1 unit, then y increases by 4 units.

Answers will vary. Three possible points are:

$x=1+1=2$ and $y=2+4=6$

$(2,6)$

$x=2+1=3$ and $y=6+4=10$

$(3,10)$

$x=3+1=4$ and $y=10+4=14$

$(4,14)$

32. Slope $= 2 = \frac{2}{1}$; point: $(-2,3)$

If x increases by 1 unit, then y increases by 2 units.

Answers will vary. Three possible points are:

$x = -2+1 = -1$ and $y = 3+2 = 5$

$(-1,5)$

$x = -1+1 = 0$ and $y = 5+2 = 7$

$(0,7)$

$x = 0+1 = 1$ and $y = 7+2 = 9$

$(1,9)$

33. Slope $= -\frac{3}{2} = \frac{-3}{2}$; point: $(2,-4)$

If x increases by 2 units, then y decreases by 3 units.

Answers will vary. Three possible points are:

$x = 2+2 = 4$ and $y = -4-3 = -7$

$(4,-7)$

$x = 4+2 = 6$ and $y = -7-3 = -10$

$(6,-10)$

$x = 6+2 = 8$ and $y = -10-3 = -13$

$(8,-13)$

34. Slope $= \frac{4}{3}$; point: $(-3,2)$

If x increases by 3 units, then y increases by 4 units.

Answers will vary. Three possible points are:

$x = -3+3 = 0$ and $y = 2+4 = 6$

$(0,6)$

$x = 0+3 = 3$ and $y = 6+4 = 10$

$(3,10)$

$x = 3+3 = 6$ and $y = 10+4 = 14$

$(6,14)$

35. Slope $= -2 = \frac{-2}{1}$; point: $(-2,-3)$

If x increases by 1 unit, then y decreases by 2 units.

Answers will vary. Three possible points are:

$x = -2+1 = -1$ and $y = -3-2 = -5$

$(-1,-5)$

$x = -1+1 = 0$ and $y = -5-2 = -7$

$(0,-7)$

$x = 0+1 = 1$ and $y = -7-2 = -9$

$(1,-9)$

36. Slope $= -1 = \frac{-1}{1}$; point: $(4,1)$

If x increases by 1 unit, then y decreases by 1 unit.

Answers will vary. Three possible points are:

$x = 4+1 = 5$ and $y = 1-1 = 0$

$(5,0)$

$x = 5+1 = 6$ and $y = 0-1 = -1$

$(6,-1)$

$x = 6+1 = 7$ and $y = -1-1 = -2$

$(7,-2)$

37. (0, 0) and (2, 1) are points on the line.

Slope $= \frac{1-0}{2-0} = \frac{1}{2}$

y-intercept is 0; using $y = mx+b$:

$$y = \frac{1}{2}x + 0$$
$$2y = x$$
$$0 = x - 2y$$

$x - 2y = 0$ or $y = \frac{1}{2}x$

38. (0, 0) and (–2, 1) are points on the line.

Slope $= \frac{1-0}{-2-0} = \frac{1}{-2} = -\frac{1}{2}$

y-intercept is 0; using $y = mx+b$:

$$y = -\frac{1}{2}x + 0$$
$$2y = -x$$
$$x + 2y = 0$$

$x + 2y = 0$ or $y = -\frac{1}{2}x$

39. (–1, 3) and (1, 1) are points on the line.

$$\text{Slope} = \frac{1-3}{1-(-1)} = \frac{-2}{2} = -1$$

Using $y - y_1 = m(x - x_1)$

$$y-1=-1(x-1)$$
$$y-1=-x+1$$
$$y=-x+2$$
$$x+y=2 \text{ or } y=-x+2$$

40. (–1, 1) and (2, 2) are points on the line.

$$\text{Slope} = \frac{2-1}{2-(-1)} = \frac{1}{3}$$

Using $y - y_1 = m(x - x_1)$

$$y-1=\frac{1}{3}\left(x-(-1)\right)$$
$$y-1=\frac{1}{3}(x+1)$$
$$y-1=\frac{1}{3}x+\frac{1}{3}$$
$$y=\frac{1}{3}x+\frac{4}{3}$$
$$x-3y=-4 \text{ or } y=\frac{1}{3}x+\frac{4}{3}$$

41. $y-y_1=m(x-x_1), \ m=2$

$$y-3=2(x-3)$$
$$y-3=2x-6$$
$$y=2x-3$$
$$2x-y=3 \text{ or } y=2x-3$$

42. $y-y_1=m(x-x_1), \ m=-1$

$$y-2=-1(x-1)$$
$$y-2=-x+1$$
$$y=-x+3$$
$$x+y=3 \text{ or } y=-x+3$$

43. $y-y_1=m(x-x_1), \ m=-\frac{1}{2}$

$$y-2=-\frac{1}{2}(x-1)$$
$$y-2=-\frac{1}{2}x+\frac{1}{2}$$
$$y=-\frac{1}{2}x+\frac{5}{2}$$
$$x+2y=5 \text{ or } y=-\frac{1}{2}x+\frac{5}{2}$$

44. $y-y_1=m(x-x_1), \ m=1$

$$y-1=1(x-(-1))$$
$$y-1=x+1$$
$$y=x+2$$
$$x-y=-2 \text{ or } y=x+2$$

45. Slope = 3; containing (–2, 3)

$$y-y_1=m(x-x_1)$$
$$y-3=3(x-(-2))$$
$$y-3=3x+6$$
$$y=3x+9$$
$$3x-y=-9 \text{ or } y=3x+9$$

46. Slope = 2; containing the point (4, –3)

$$y-y_1=m(x-x_1)$$
$$y-(-3)=2(x-4)$$
$$y+3=2x-8$$
$$y=2x-11$$
$$2x-y=11 \text{ or } y=2x-11$$

47. Slope $=-\frac{2}{3}$; containing (1, –1)

$$y-y_1=m(x-x_1)$$
$$y-(-1)=-\frac{2}{3}(x-1)$$
$$y+1=-\frac{2}{3}x+\frac{2}{3}$$
$$y=-\frac{2}{3}x-\frac{1}{3}$$
$$2x+3y=-1 \text{ or } y=-\frac{2}{3}x-\frac{1}{3}$$

48. Slope $=\frac{1}{2}$; containing the point (3, 1)

$$y-y_1=m(x-x_1)$$
$$y-1=\frac{1}{2}(x-3)$$
$$y-1=\frac{1}{2}x-\frac{3}{2}$$
$$y=\frac{1}{2}x-\frac{1}{2}$$
$$x-2y=1 \text{ or } y=\frac{1}{2}x-\frac{1}{2}$$

49. Containing (1, 3) and (–1, 2)

$$m=\frac{2-3}{-1-1}=\frac{-1}{-2}=\frac{1}{2}$$

$$y-y_1=m(x-x_1)$$

$$y-3=\frac{1}{2}(x-1)$$

$$y-3=\frac{1}{2}x-\frac{1}{2}$$

$$y=\frac{1}{2}x+\frac{5}{2}$$

$x-2y=-5$ or $y=\frac{1}{2}x+\frac{5}{2}$

50. Containing the points (–3, 4) and (2, 5)

$$m=\frac{5-4}{2-(-3)}=\frac{1}{5}$$

$$y-y_1=m(x-x_1)$$

$$y-5=\frac{1}{5}(x-2)$$

$$y-5=\frac{1}{5}x-\frac{2}{5}$$

$$y=\frac{1}{5}x+\frac{23}{5}$$

$x-5y=-23$ or $y=\frac{1}{5}x+\frac{23}{5}$

51. Slope = –3; y-intercept =3

$$y=mx+b$$

$$y=-3x+3$$

$3x+y=3$ or $y=-3x+3$

52. Slope = –2; y-intercept =–2

$$y=mx+b$$

$$y=-2x+(-2)$$

$$y=-2x-2$$

$2x+y=-2$ or $y=-2x-2$

53. x-intercept = 2; y-intercept = –1
Points are (2,0) and (0,–1)

$$m=\frac{-1-0}{0-2}=\frac{-1}{-2}=\frac{1}{2}$$

$$y=mx+b$$

$$y=\frac{1}{2}x-1$$

$x-2y=2$ or $y=\frac{1}{2}x-1$

54. x-intercept = –4; y-intercept = 4
Points are (–4, 0) and (0, 4)

$$m=\frac{4-0}{0-(-4)}=\frac{4}{4}=1$$

$$y=mx+b$$

$$y=1x+4$$

$$y=x+4$$

$x-y=-4$ or $y=x+4$

55. Slope undefined; containing the point (2, 4)
This is a vertical line.
$x=2$ No slope intercept form.

56. Slope undefined; containing the point (3, 8)
This is a vertical line.
$x=3$ No slope intercept form.

57. Horizontal lines have slope $m=0$ and take the form $y=b$. Therefore, the horizontal line passing through the point $(-3,2)$ is $y=2$.

58. Vertical lines have an undefined slope and take the form $x=a$. Therefore, the vertical line passing through the point $(4,-5)$ is $x=4$.

59. Parallel to $y=2x$; Slope = 2
Containing (–1, 2)

$$y-y_1=m(x-x_1)$$

$$y-2=2(x-(-1))$$

$$y-2=2x+2\rightarrow y=2x+4$$

$2x-y=-4$ or $y=2x+4$

60. Parallel to $y=-3x$; Slope = –3; Containing the point (–1, 2)

$$y-y_1=m(x-x_1)$$

$$y-2=-3(x-(-1))$$

$$y-2=-3x-3\rightarrow y=-3x-1$$

$3x+y=-1$ or $y=-3x-1$

61. Parallel to $2x-y=-2$; Slope = 2
Containing the point (0, 0)

$$y-y_1=m(x-x_1)$$

$$y-0=2(x-0)$$

$$y=2x$$

$2x-y=0$ or $y=2x$

62. Parallel to $x-2y=-5$;

Slope$=\frac{1}{2}$; Containing the point $(0,0)$

$y-y_1=m(x-x_1)$

$y-0=\frac{1}{2}(x-0)\rightarrow y=\frac{1}{2}x$

$x-2y=0$ or $y=\frac{1}{2}x$

63. Parallel to $x=5$; Containing (4,2)
This is a vertical line.
$x=4$ No slope intercept form.

64. Parallel to $y=5$; Containing the point (4, 2)
This is a horizontal line. Slope = 0
$y=2$

65. Perpendicular to $y=\frac{1}{2}x+4$; Containing (1, –2)

Slope of perpendicular = –2

$y-y_1=m(x-x_1)$

$y-(-2)=-2(x-1)$

$y+2=-2x+2\rightarrow y=-2x$

$2x+y=0$ or $y=-2x$

66. Perpendicular to $y=2x-3$; Containing the point (1, –2)

Slope of perpendicular $=-\frac{1}{2}$

$y-y_1=m(x-x_1)$

$y-(-2)=-\frac{1}{2}(x-1)$

$y+2=-\frac{1}{2}x+\frac{1}{2}\rightarrow y=-\frac{1}{2}x-\frac{3}{2}$

$x+2y=-3$ or $y=-\frac{1}{2}x-\frac{3}{2}$

67. Perpendicular to $2x+y=2$; Containing the point (–3, 0)

Slope of perpendicular $=\frac{1}{2}$

$y-y_1=m(x-x_1)$

$y-0=\frac{1}{2}(x-(-3))\rightarrow y=\frac{1}{2}x+\frac{3}{2}$

$x-2y=-3$ or $y=\frac{1}{2}x+\frac{3}{2}$

68. Perpendicular to $x-2y=-5$; Containing the point (0, 4)
Slope of perpendicular = –2

$y=mx+b$

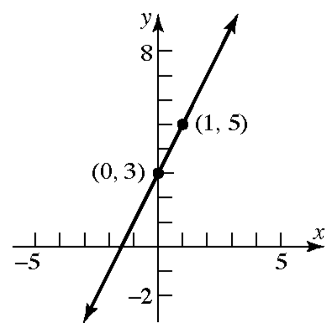
$y=-2x+4$

$2x+y=4$ or $y=-2x+4$

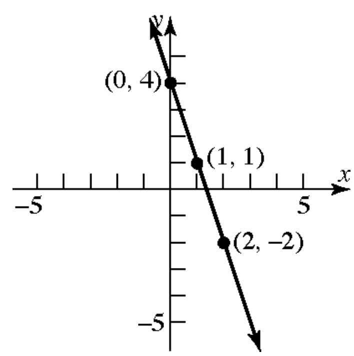
69. Perpendicular to $x=8$; Containing (3, 4)
Slope of perpendicular = 0 (horizontal line)
$y=4$

70. Perpendicular to $y=8$;
Containing the point (3, 4)
Slope of perpendicular is undefined (vertical line). $x=3$ No slope-intercept form.

71. $y=2x+3$; Slope = 2; y-intercept = 3

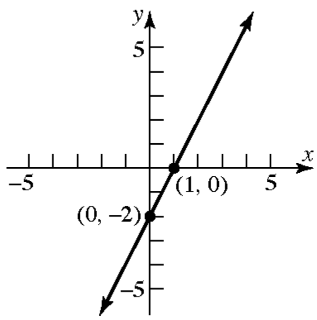


72. $y=-3x+4$; Slope = –3; y-intercept = 4



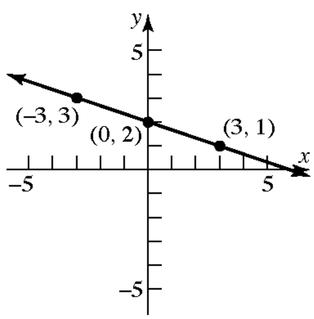
73. $\frac{1}{2}y = x - 1$; $y = 2x - 2$

Slope = 2; y-intercept = –2

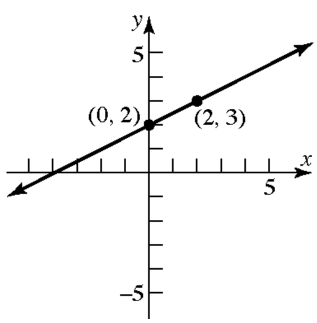


74. $\frac{1}{3}x + y = 2$; $y = -\frac{1}{3}x + 2$

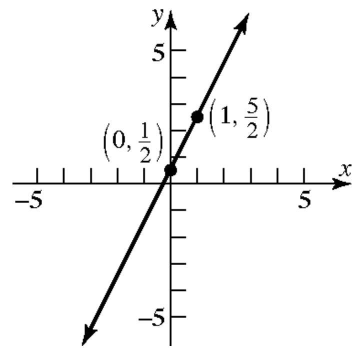
Slope $= -\frac{1}{3}$; y-intercept = 2



75. $y = \frac{1}{2}x + 2$; Slope$=\frac{1}{2}$; y-intercept = 2

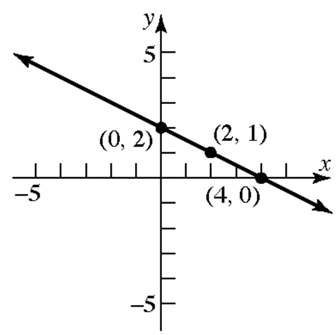


76. $y = 2x + \frac{1}{2}$; Slope = 2; y-intercept$=\frac{1}{2}$



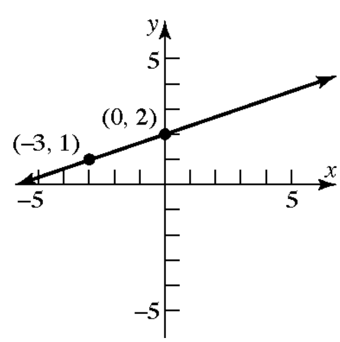
77. $x + 2y = 4$; $2y = -x + 4 \rightarrow y = -\frac{1}{2}x + 2$

Slope $= -\frac{1}{2}$; y-intercept = 2



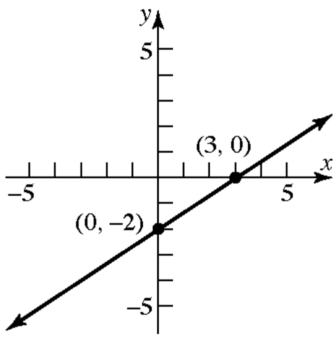
78. $-x + 3y = 6$; $3y = x + 6 \rightarrow y = \frac{1}{3}x + 2$

Slope $= \frac{1}{3}$; y-intercept = 2



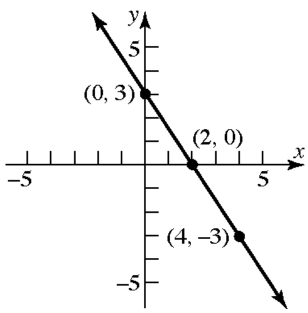
79. $2x-3y=6$; $-3y=-2x+6 \rightarrow y=\frac{2}{3}x-2$

Slope $=\frac{2}{3}$; y-intercept = –2

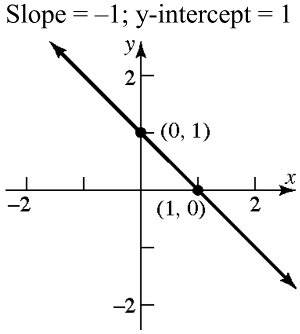


80. $3x+2y=6$; $2y=-3x+6 \rightarrow y=-\frac{3}{2}x+3$

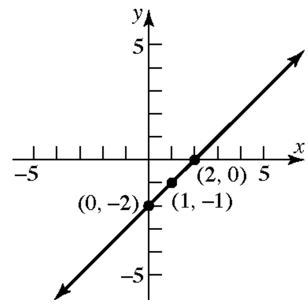
Slope $=-\frac{3}{2}$; y-intercept = 3



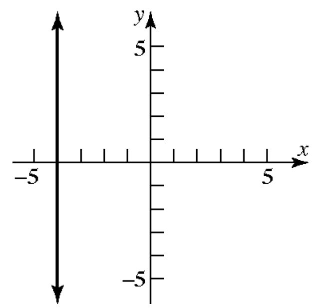
81. $x+y=1$; $y=-x+1$
Slope = –1; y-intercept = 1



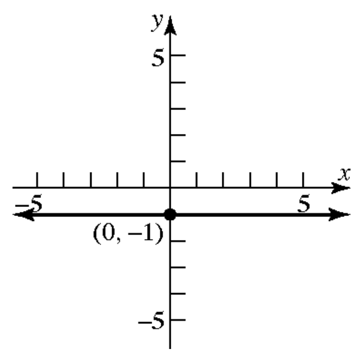
82. $x-y=2$; $y=x-2$
Slope = 1; y-intercept = –2



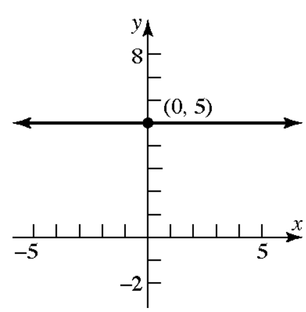
83. $x=-4$; Slope is undefined
y-intercept - none



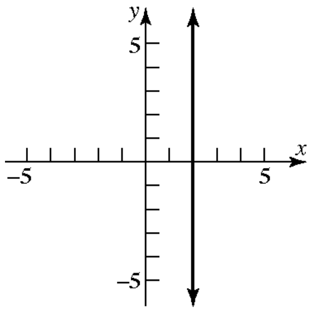
84. $y=-1$; Slope = 0; y-intercept = –1



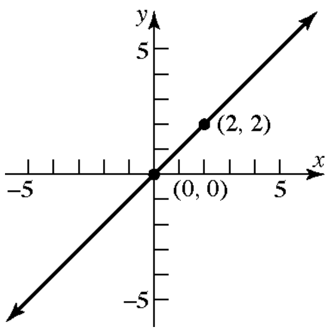
85. $y=5$; Slope = 0; y-intercept = 5



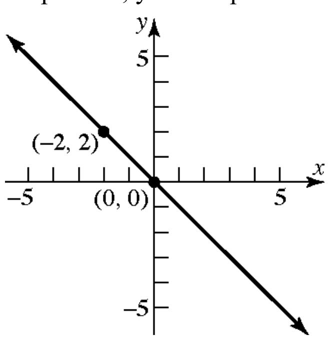
86. $x = 2$; Slope is undefined
y-intercept - none



87. $y - x = 0$; $y = x$
Slope = 1; y-intercept = 0

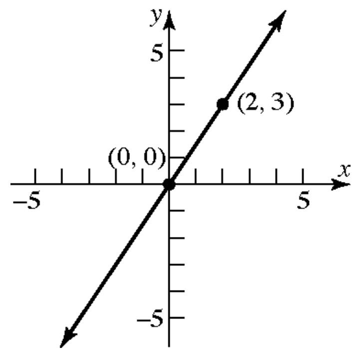


88. $x + y = 0$; $y = -x$
Slope = –1; y-intercept = 0



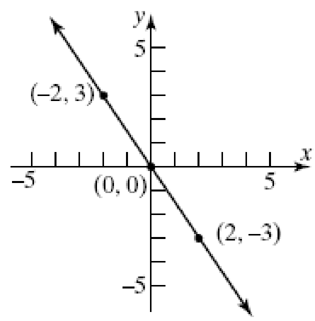
89. $2y - 3x = 0$; $2y = 3x \rightarrow y = \frac{3}{2}x$

Slope = $\frac{3}{2}$; y-intercept = 0



90. $3x + 2y = 0$; $2y = -3x \rightarrow y = -\frac{3}{2}x$

Slope = $-\frac{3}{2}$; y-intercept = 0



91. a. x-intercept: $2x + 3(0) = 6$

$$2x = 6$$
$$x = 3$$

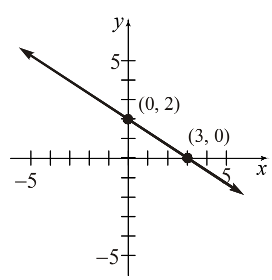
The point $(3, 0)$ is on the graph.

y-intercept: $2(0) + 3y = 6$

$$3y = 6$$
$$y = 2$$

The point $(0, 2)$ is on the graph.

b.



92. a. x-intercept: $3x - 2(0) = 6$

$$3x = 6$$
$$x = 2$$

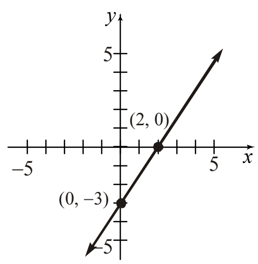
The point $(2, 0)$ is on the graph.

y-intercept: $3(0) - 2y = 6$

$$-2y = 6$$
$$y = -3$$

The point $(0, -3)$ is on the graph.

b.



93. a. x-intercept: $-4x + 5(0) = 40$

$$-4x = 40$$
$$x = -10$$

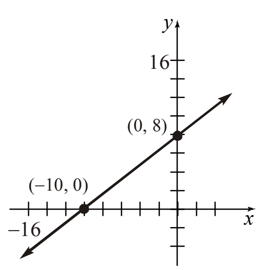
The point $(-10, 0)$ is on the graph.

y-intercept: $-4(0) + 5y = 40$

$$5y = 40$$
$$y = 8$$

The point $(0, 8)$ is on the graph.

b.



94. a. x-intercept: $6x - 4(0) = 24$

$$6x = 24$$
$$x = 4$$

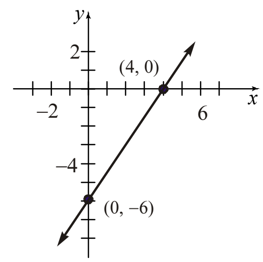
The point $(4, 0)$ is on the graph.

y-intercept: $6(0) - 4y = 24$

$$-4y = 24$$
$$y = -6$$

The point $(0, -6)$ is on the graph.

b.



95. a. x-intercept: $7x + 2(0) = 21$

$$7x = 21$$
$$x = 3$$

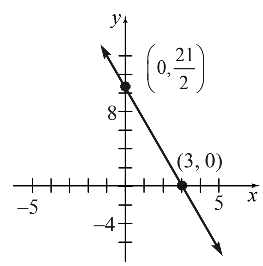
The point $(3, 0)$ is on the graph.

y-intercept: $7(0) + 2y = 21$

$$2y = 21$$
$$y = \frac{21}{2}$$

The point $\left(0, \frac{21}{2}\right)$ is on the graph.

b.



96. a. x-intercept: $5x + 3(0) = 18$

$$5x = 18$$
$$x = \frac{18}{5}$$

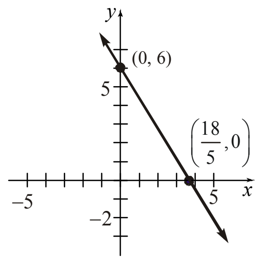
The point $\left(\frac{18}{5}, 0\right)$ is on the graph.

y-intercept: $5(0) + 3y = 18$

$$3y = 18$$
$$y = 6$$

The point $(0, 6)$ is on the graph.

b.



97. a. x-intercept: $\frac{1}{2}x + \frac{1}{3}(0) = 1$

$$\frac{1}{2}x = 1$$
$$x = 2$$

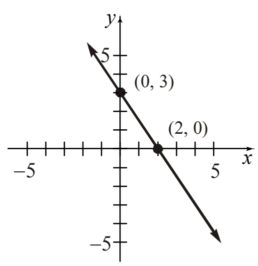
The point $(2, 0)$ is on the graph.

y-intercept: $\frac{1}{2}(0) + \frac{1}{3}y = 1$

$$\frac{1}{3}y = 1$$
$$y = 3$$

The point $(0, 3)$ is on the graph.

b.



98. a. x-intercept: $x - \frac{2}{3}(0) = 4$

$$x = 4$$

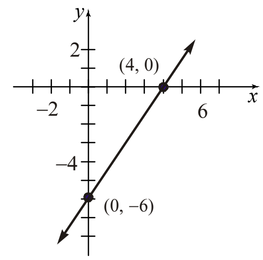
The point $(4, 0)$ is on the graph.

y-intercept: $(0) - \frac{2}{3}y = 4$

$$-\frac{2}{3}y = 4$$
$$y = -6$$

The point $(0, -6)$ is on the graph.

b.



99. a. x-intercept: $0.2x - 0.5(0) = 1$

$$0.2x = 1$$
$$x = 5$$

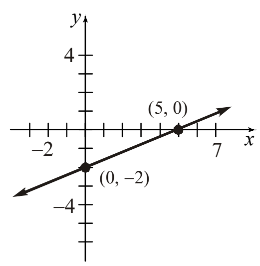
The point $(5, 0)$ is on the graph.

y-intercept: $0.2(0) - 0.5y = 1$

$$-0.5y = 1$$
$$y = -2$$

The point $(0, -2)$ is on the graph.

b.



100. a. x-intercept: $-0.3x + 0.4(0) = 1.2$

$$-0.3x = 1.2$$
$$x = -4$$

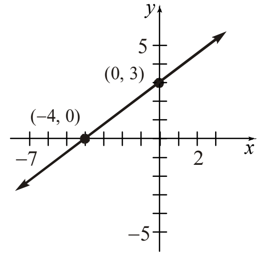
The point $(-4, 0)$ is on the graph.

y-intercept: $-0.3(0) + 0.4y = 1.2$

$$0.4y = 1.2$$
$$y = 3$$

The point $(0, 3)$ is on the graph.

b.



101. The equation of the x-axis is $y=0$. (The slope is 0 and the y-intercept is 0.)

102. The equation of the y-axis is $x=0$. (The slope is undefined.)

103. The slopes are the same but the *y*-intercepts are different. Therefore, the two lines are parallel.

104. The slopes are opposite-reciprocals. That is, their product is -1. Therefore, the lines are perpendicular.

105. The slopes are different and their product does not equal -1. Therefore, the lines are neither parallel nor perpendicular.

106. The slopes are different and their product does not equal -1 (in fact, the signs are the same so the product is positive). Therefore, the lines are neither parallel nor perpendicular.

107. Intercepts: $(0,2)$ and $(-2,0)$. Thus, slope = 1.

$y=x+2$ or $x-y=-2$

108. Intercepts: $(0,1)$ and $(1,0)$. Thus, slope = –1.

$y=-x+1$ or $x+y=1$

109. Intercepts: $(3,0)$ and $(0,1)$. Thus, slope = $-\frac{1}{3}$.

$y=-\frac{1}{3}x+1$ or $x+3y=3$

110. Intercepts: $(0,-1)$ and $(-2,0)$. Thus, slope = $-\frac{1}{2}$.

$y=-\frac{1}{2}x-1$ or $x+2y=-2$

111. $P_1(-2,5)$, $P_2(1,3)$: $m_1=\frac{5-3}{-2-1}=\frac{2}{-3}=-\frac{2}{3}$

$P_2(1,3)$, $P_3(-1,0)$: $m_2=\frac{3-0}{1-(-1)}=\frac{3}{2}$

Since $m_1\cdot m_2=-1$, the line segments $\overline{P_1P_2}$ and $\overline{P_2P_3}$ are perpendicular. Thus, the points P_1, P_2, and P_3 are vertices of a right triangle.

112. $P_1(1,-1)$, $P_2(4,1)$, $P_3(2,2)$, $P_4(5,4)$

$m_{12}=\frac{1-(-1)}{4-1}=\frac{2}{3}$; $m_{24}=\frac{4-1}{5-4}=3$;

$m_{34}=\frac{4-2}{5-2}=\frac{2}{3}$; $m_{13}=\frac{2-(-1)}{2-1}=3$

Each pair of opposite sides are parallel (same slope) and adjacent sides are not perpendicular. Therefore, the vertices are for a parallelogram.

113. $P_1(-1,0)$, $P_2(2,3)$, $P_3(1,-2)$, $P_4(4,1)$

$m_{12}=\frac{3-0}{2-(-1)}=\frac{3}{3}=1$; $m_{24}=\frac{1-3}{4-2}=-1$;

$m_{34}=\frac{1-(-2)}{4-1}=\frac{3}{3}=1$; $m_{13}=\frac{-2-0}{1-(-1)}=-1$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). Therefore, the vertices are for a rectangle.

114. $P_1(0,0)$, $P_2(1,3)$, $P_3(4,2)$, $P_4(3,-1)$

$m_{12}=\frac{3-0}{1-0}=3$; $m_{23}=\frac{2-3}{4-1}=-\frac{1}{3}$;

$m_{34}=\frac{-1-2}{3-4}=3$; $m_{14}=\frac{-1-0}{3-0}=-\frac{1}{3}$

$d_{12}=\sqrt{(1-0)^2+(3-0)^2}=\sqrt{1+9}=\sqrt{10}$

$d_{23}=\sqrt{(4-1)^2+(2-3)^2}=\sqrt{9+1}=\sqrt{10}$

$d_{34}=\sqrt{(3-4)^2+(-1-2)^2}=\sqrt{1+9}=\sqrt{10}$

$d_{14}=\sqrt{(3-0)^2+(-1-0)^2}=\sqrt{9+1}=\sqrt{10}$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). In addition, the length of all four sides is the same. Therefore, the vertices are for a square.

115. Let *x* = number of miles driven, and let *C* = cost in dollars.
Total cost = (cost per mile)(number of miles) + fixed cost
$C=0.07x+29$
When *x* = 110, $C=(0.07)(110)+29=\$36.70$.
When *x* = 230, $C=(0.07)(230)+29=\$45.10$.

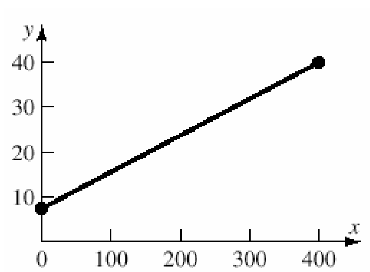
116. Let x = number of pairs of jeans manufactured, and let C = cost in dollars.
Total cost = (cost per pair)(number of pairs) + fixed cost
$C = 8x + 500$
When x = 400, $C = (8)(400) + 500 = \$3700$.
When x = 740, $C = (8)(740) + 500 = \$6420$.

117. Let x = number newspapers delivered, and let C = cost in dollars.
Total cost = (delivery cost per paper)(number of papers delivered) + fixed cost
$C = 0.53x + 1,070,000$

118. Let x = profit in dollars, and let S = salary in dollars.
Weekly salary = (% share of profit)(profit) + weekly pay
$S = 0.05x + 375$

119. **a.** $C = 0.08275x + 7.58$; $0 \le x \le 400$

b.



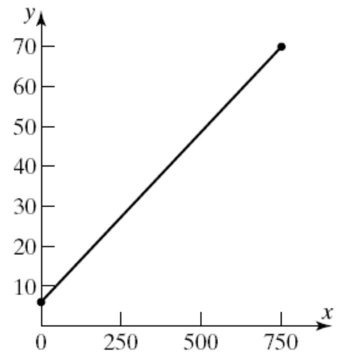
c. For 100 kWh,
$C = 0.08275(100) + 7.58 = \$15.86$

d. For 300 kWh,
$C = 0.08725(300) + 7.58 = \$32.41$

e. For each usage increase of 1 kWh, the monthly charge increases by 8.275 cents

120. **a.** $C = 0.0835x + 5.25$; $0 \le x \le 750$

b.



c. For 200 kWh,
$C = 0.0835(200) + 5.25 = \$21.95$

d. For 500 kWh,
$C = 0.0835(500) + 5.25 = \$47.00$

e. For each usage increase of 1 kWh, the monthly charge increases by 8.35 cents

121. $(°C, °F) = (0, 32)$; $(°C, °F) = (100, 212)$

$$\text{slope} = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$$

$$°F - 32 = \frac{9}{5}(°C - 0)$$

$$°F - 32 = \frac{9}{5}(°C)$$

$$°C = \frac{5}{9}(°F - 32)$$

If $°F = 70$, then

$$°C = \frac{5}{9}(70-32) = \frac{5}{9}(38)$$

$$°C \approx 21.1°$$

122. **a.** $K = °C + 273$

b. $°C = \frac{5}{9}(°F - 32)$

$$K = \frac{5}{9}(°F - 32) + 273$$

$$K = \frac{5}{9}°F - \frac{160}{9} + 273$$

$$K = \frac{5}{9}°F + \frac{2297}{9}$$

$$K = \frac{1}{9}(5°F + 2297)$$

123. **a.** Let x = number of boxes sold, and A = money, in dollars, spent on advertising.
We have the points
$(x_1, A_1) = (100,000, 40,000)$;
$(x_2, A_2) = (200,000, 60,000)$

$$\text{slope} = \frac{60,000-40,000}{200,000-100,000} = \frac{20,000}{100,000} = \frac{1}{5}$$

$$A - 40,000 = \frac{1}{5}(x - 100,000)$$

$$A - 40,000 = \frac{1}{5}x - 20,000$$

$$A = \frac{1}{5}x + 20,000$$

b. If x = 300,000, then

$$A = \frac{1}{5}(300,000) + 20,000 = \$80,000$$

c. Each additional box sold requires an additional $0.20 in advertising.

124. Find the slope of the line containing (a,b) and (b,a):

$$\text{slope} = \frac{a-b}{b-a} = -1$$

The slope of the line $y = x$ is 1.

Since $-1 \cdot 1 = -1$, the line containing the points (a,b) and (b,a) is perpendicular to the line $y = x$.

The midpoint of (a,b) and (b,a) is

$$M = \left(\frac{a+b}{2}, \frac{b+a}{2}\right).$$

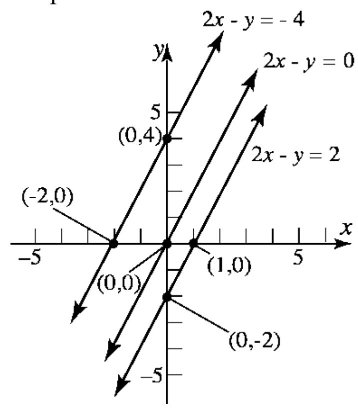
Since the coordinates are the same, the midpoint lies on the line $y = x$.

Note: $\frac{a+b}{2} = \frac{b+a}{2}$

125. $2x - y = C$

Graph the lines:
$2x - y = -4$
$2x - y = 0$
$2x - y = 2$

All the lines have the same slope, 2. The lines are parallel.



126. Refer to Figure 45.

length of $\overline{OA} = \sqrt{1+m_1^2}$

length of $\overline{OB} = \sqrt{1+m_2^2}$

length of $\overline{AB} = m_1 - m_2$

Now consider the equation

$$\left(\sqrt{1+m_1^2}\right)^2 + \left(\sqrt{1+m_2^2}\right)^2 = (m_1 - m_2)^2$$

If this equation is valid, then ΔAOB is a right triangle with right angle at vertex O.

$$\left(\sqrt{1+m_1^2}\right)^2 + \left(\sqrt{1+m_2^2}\right)^2 = (m_1 - m_2)^2$$

$$1 + m_1^2 + 1 + m_2^2 = m_1^2 - 2m_1m_2 + m_2^2$$

$$2 + m_1^2 + m_2^2 = m_1^2 - 2m_1m_2 + m_2^2$$

But we are assuming that $m_1m_2 = -1$, so we have

$$2 + m_1^2 + m_2^2 = m_1^2 - 2(-1) + m_2^2$$

$$2 + m_1^2 + m_2^2 = m_1^2 + 2 + m_2^2$$

$$0 = 0$$

Therefore, by the converse of the Pythagorean Theorem, ΔAOB is a right triangle with right angle at vertex O. Thus Line 1 is perpendicular to Line 2.

127. (b), (c), (e) and (g)
The line has positive slope and positive y-intercept.

128. (a), (c), and (g)
The line has negative slope and positive y-intercept.

129. (c)

130. (d)

131. -133. Answers will vary.

134. No, the equation of a vertical line cannot be written in slope-intercept form because the slope is undefined.

135. No, a line does not need to have both an x-intercept and a y-intercept. Vertical and horizontal lines have only one intercept (unless they are a coordinate axis). Every line must have at least one intercept.

136. Two lines with equal slopes and equal y-intercepts are coinciding lines (i.e. the same).

137. Two lines that have the same x-intercept and y-intercept (assuming the x-intercept is not 0) are the same line since a line is uniquely defined by two distinct points.

138. No; two lines with the same slope and different x-intercepts are distinct parallel lines and have no points in common.
Assume Line 1 has equation $y = mx + b_1$ and Line 2 has equation $y = mx + b_2$,
Line 1 has x-intercept $-\frac{b_1}{m}$ and y-intercept b_1.
Line 2 has x-intercept $-\frac{b_2}{m}$ and y-intercept b_2.
Assume also that Line 1 and Line 2 have unequal x-intercepts.
If the lines have the same y-intercept, then $b_1 = b_2$.

$$b_1 = b_2 \Rightarrow \frac{b_1}{m} = \frac{b_2}{m} \Rightarrow -\frac{b_1}{m} = -\frac{b_2}{m}$$

But $-\frac{b_1}{m} = -\frac{b_2}{m} \Rightarrow$ Line 1 and Line 2 have the same x-intercept, which contradicts the original assumption that the lines have unequal x-intercepts.
Therefore, Line 1 and Line 2 cannot have the same y-intercept.

139. Yes; two lines with the same y-intercept, but different slopes, can have the same x-intercept if the x-intercept is $x = 0$.
Assume Line 1 has equation $y = m_1 x + b$ and Line 2 has equation $y = m_2 x + b$,
Line 1 has x-intercept $-\frac{b}{m_1}$ and y-intercept b.
Line 2 has x-intercept $-\frac{b}{m_2}$ and y-intercept b.
Assume also that Line 1 and Line 2 have unequal slopes, that is $m_1 \neq m_2$.
If the lines have the same x-intercept, then $-\frac{b}{m_1} = -\frac{b}{m_2}$.

$$-\frac{b}{m_1} = -\frac{b}{m_2}$$
$$-m_2 b = -m_1 b$$
$$-m_2 b + m_1 b = 0$$

But $-m_2 b + m_1 b = 0 \Rightarrow b(m_1 - m_2) = 0$
$\Rightarrow b = 0$
or $m_1 - m_2 = 0 \Rightarrow m_1 = m_2$

Since we are assuming that $m_1 \neq m_2$, the only way that the two lines can have the same x-intercept is if $b = 0$.

140. Answers will vary.

Section F.4

1. add; 25

2. $\left(\frac{-7}{2}\right)^2 = \frac{49}{4}$

   $x^2 - 7x + \frac{49}{4} = \left(x - \frac{7}{2}\right)^2$

3. False; for example, $x^2 + y^2 + 2x + 2y + 8 = 0$ is not a circle – it has no real solutions.

4. radius

5. True; $r^2 = 9 \rightarrow r = 3$

6. False; the center of the circle $(x+3)^2 + (y-2)^2 = 13$ is $(-3, 2)$.

7. Center = (2, 1)
   Radius = distance from (0,1) to (2,1)
   $= \sqrt{(2-0)^2 + (1-1)^2} = \sqrt{4} = 2$
   Equation: $(x-2)^2 + (y-1)^2 = 4$

8. Center = (1, 2)

Radius = distance from (1,0) to (1,2)

$$= \sqrt{(1-1)^2 + (2-0)^2} = \sqrt{4} = 2$$

Equation: $(x-1)^2 + (y-2)^2 = 4$

9. Center = midpoint of (1, 2) and (4, 2)

$$= \left(\frac{1+4}{2}, \frac{2+2}{2}\right) = \left(\frac{5}{2}, 2\right)$$

Radius = distance from $\left(\frac{5}{2}, 2\right)$ to (4,2)

$$= \sqrt{\left(4-\frac{5}{2}\right)^2 + (2-2)^2} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

Equation: $\left(x-\frac{5}{2}\right)^2 + (y-2)^2 = \frac{9}{4}$

10. Center = midpoint of (0, 1) and (2, 3)

$$= \left(\frac{0+2}{2}, \frac{1+3}{2}\right) = (1, 2)$$

Radius = distance from $(1,2)$ to (2,3)

$$= \sqrt{(2-1)^2 + (3-2)^2} = \sqrt{2}$$

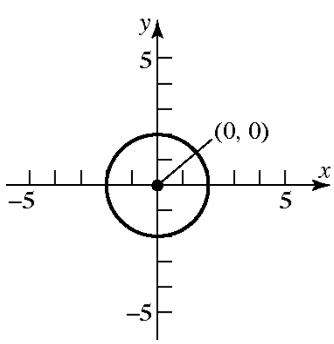
Equation: $(x-1)^2 + (y-2)^2 = 2$

11. $(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-0)^2 = 2^2$

$x^2 + y^2 = 4$

General form: $x^2 + y^2 - 4 = 0$

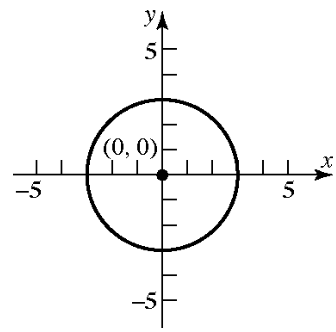


12. $(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-0)^2 = 3^2$

$x^2 + y^2 = 9$

General form: $x^2 + y^2 - 9 = 0$



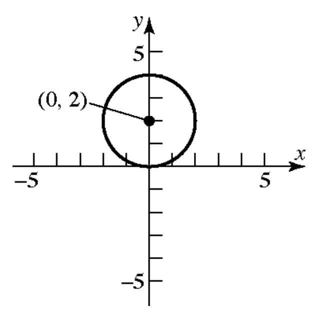
13. $(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-2)^2 = 2^2$

$x^2 + (y-2)^2 = 4$

General form: $x^2 + y^2 - 4y + 4 = 4$

$x^2 + y^2 - 4y = 0$



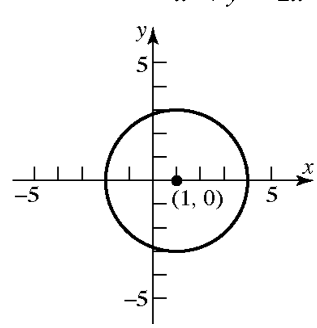
14. $(x-h)^2 + (y-k)^2 = r^2$

$(x-1)^2 + (y-0)^2 = 3^2$

$(x-1)^2 + y^2 = 9$

General form: $x^2 - 2x + 1 + y^2 = 9$

$x^2 + y^2 - 2x - 8 = 0$



15. $(x-h)^2+(y-k)^2=r^2$

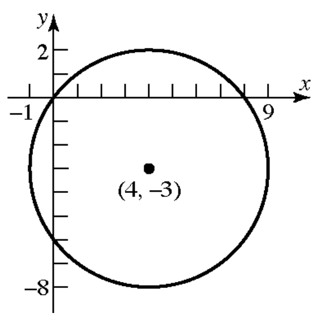
$(x-4)^2+(y-(-3))^2=5^2$

$(x-4)^2+(y+3)^2=25$

General form:

$x^2-8x+16+y^2+6y+9=25$

$x^2+y^2-8x+6y=0$



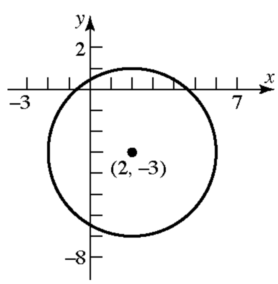
16. $(x-h)^2+(y-k)^2=r^2$

$(x-2)^2+(y-(-3))^2=4^2$

$(x-2)^2+(y+3)^2=16$

General form: $x^2-4x+4+y^2+6y+9=16$

$x^2+y^2-4x+6y-3=0$



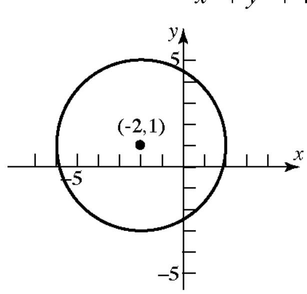
17. $(x-h)^2+(y-k)^2=r^2$

$(x-(-2))^2+(y-1)^2=4^2$

$(x+2)^2+(y-1)^2=16$

General form: $x^2+4x+4+y^2-2y+1=16$

$x^2+y^2+4x-2y-11=0$



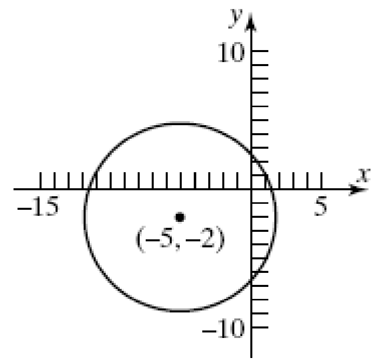
18. $(x-h)^2+(y-k)^2=r^2$

$(x-(-5))^2+(y-(-2))^2=7^2$

$(x+5)^2+(y+2)^2=49$

General form: $x^2+10x+25+y^2+4y+4=49$

$x^2+y^2+10x+4y-20=0$



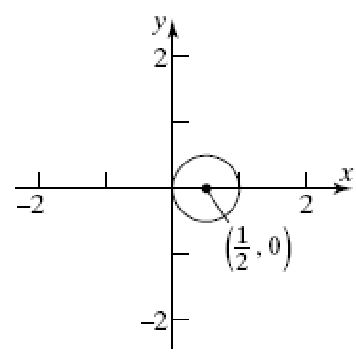
19. $(x-h)^2+(y-k)^2=r^2$

$\left(x-\frac{1}{2}\right)^2+(y-0)^2=\left(\frac{1}{2}\right)^2$

$\left(x-\frac{1}{2}\right)^2+y^2=\frac{1}{4}$

General form: $x^2-x+\frac{1}{4}+y^2=\frac{1}{4}$

$x^2+y^2-x=0$



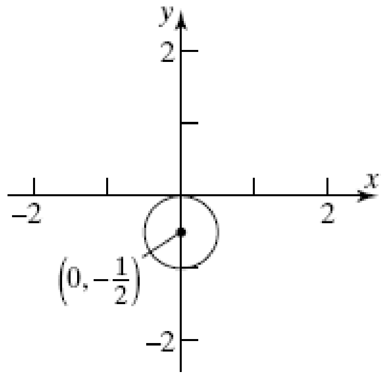
20. $(x-h)^2+(y-k)^2=r^2$

$(x-0)^2+\left(y-\left(-\frac{1}{2}\right)\right)^2=\left(\frac{1}{2}\right)^2$

$x^2+\left(y+\frac{1}{2}\right)^2=\frac{1}{4}$

General form: $x^2 + y^2 + y + \frac{1}{4} = \frac{1}{4}$

$$x^2 + y^2 + y = 0$$

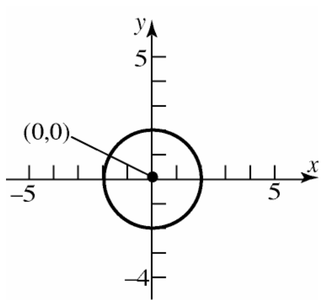


21. $x^2 + y^2 = 4$

$x^2 + y^2 = 2^2$

a. Center: $(0,0)$; Radius $= 2$

b.

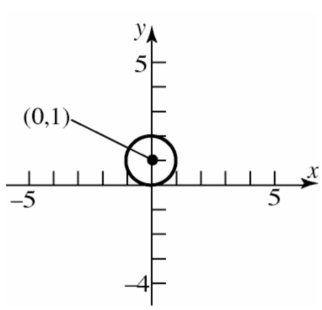


22. $x^2 + (y-1)^2 = 1$

$x^2 + (y-1)^2 = 1^2$

a. Center:$(0, 1)$; Radius $= 1$

b.

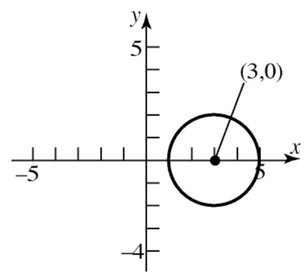


23. $2(x-3)^2 + 2y^2 = 8$

$(x-3)^2 + y^2 = 4$

a. Center: $(3, 0)$; Radius $= 2$

b.

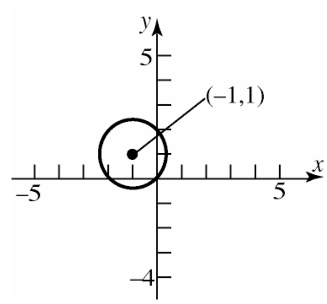


24. $3(x+1)^2 + 3(y-1)^2 = 6$

$(x+1)^2 + (y-1)^2 = 2$

a. Center: $(-1,1)$; Radius = $\sqrt{2}$

b.



25. $x^2 + y^2 - 2x - 4y - 4 = 0$

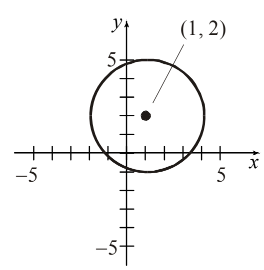
$$x^2 - 2x + y^2 - 4y = 4$$

$$(x^2 - 2x + 1) + (y^2 - 4y + 4) = 4 + 1 + 4$$

$$(x-1)^2 + (y-2)^2 = 3^2$$

a. Center: $(1, 2)$; Radius = 3

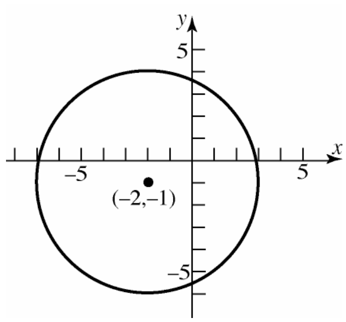
b.



26.
$$x^2+y^2+4x+2y-20=0$$
$$x^2+4x+y^2+2y=20$$
$$(x^2+4x+4)+(y^2+2y+1)=20+4+1$$
$$(x+2)^2+(y+1)^2=5^2$$

a. Center: (–2,–1); Radius = 5

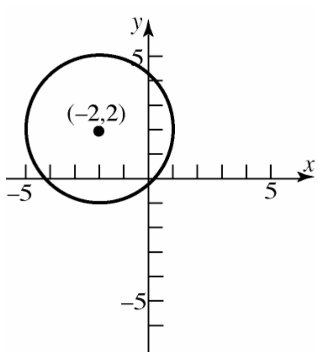
b.



27.
$$x^2+y^2+4x-4y-1=0$$
$$x^2+4x+y^2-4y=1$$
$$(x^2+4x+4)+(y^2-4y+4)=1+4+4$$
$$(x+2)^2+(y-2)^2=3^2$$

a. Center: (–2, 2); Radius = 3

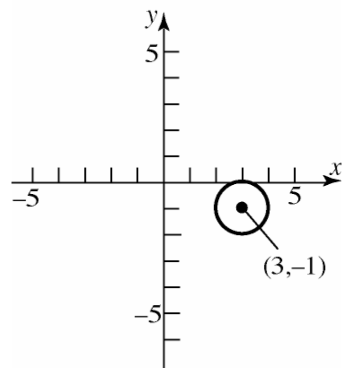
b.



28.
$$x^2+y^2-6x+2y+9=0$$
$$x^2-6x+y^2+2y=-9$$
$$(x^2-6x+9)+(y^2+2y+1)=-9+9+1$$
$$(x-3)^2+(y+1)^2=1^2$$

a. Center: (3, –1); Radius = 1

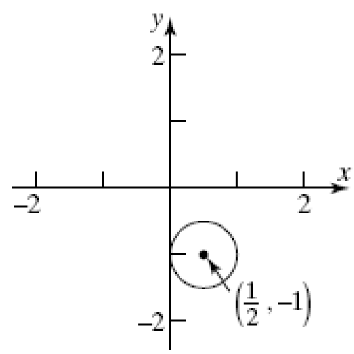
b.



29.
$$x^2+y^2-x+2y+1=0$$
$$x^2-x+y^2+2y=-1$$
$$\left(x^2-x+\frac{1}{4}\right)+(y^2+2y+1)=-1+\frac{1}{4}+1$$
$$\left(x-\frac{1}{2}\right)^2+(y+1)^2=\left(\frac{1}{2}\right)^2$$

a. Center: $\left(\frac{1}{2},-1\right)$; Radius = $\frac{1}{2}$

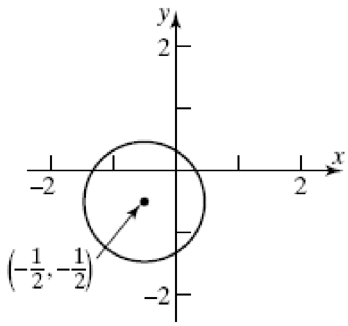
b.



30.
$$x^2+y^2+x+y-\frac{1}{2}=0$$
$$x^2+x+y^2+y=\frac{1}{2}$$
$$\left(x^2+x+\frac{1}{4}\right)+\left(y^2+y+\frac{1}{4}\right)=\frac{1}{2}+\frac{1}{4}+\frac{1}{4}$$
$$\left(x+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=1^2$$

a. Center: $\left(-\frac{1}{2},-\frac{1}{2}\right)$; Radius = 1

b.



31. $2x^2 + 2y^2 - 12x + 8y - 24 = 0$

$$x^2 + y^2 - 6x + 4y = 12$$

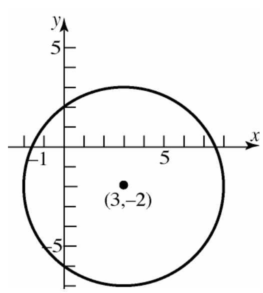
$$x^2 - 6x + y^2 + 4y = 12$$

$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4$$

$$(x-3)^2 + (y+2)^2 = 5^2$$

a. Center: (3,–2); Radius = 5

b.



32. a. $2x^2 + 2y^2 + 8x + 7 = 0$

$$x^2 + y^2 + 4x = -\frac{7}{2}$$

$$x^2 + 4x + y^2 = -\frac{7}{2}$$

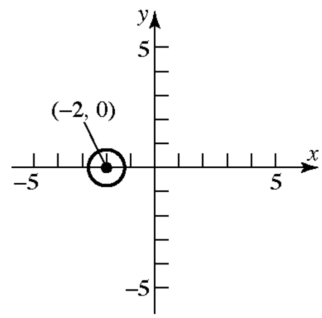
$$(x^2 + 4x + 4) + y^2 = -\frac{7}{2} + 4$$

$$(x+2)^2 + y^2 = \frac{1}{2}$$

$$(x+2)^2 + y^2 = \left(\frac{\sqrt{2}}{2}\right)^2$$

Center: (–2, 0); Radius = $\frac{\sqrt{2}}{2}$

b.



33. $2x^2 + 4x + 2y^2 = 0$

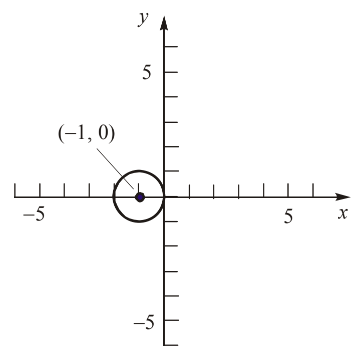
$$x^2 + 2x + y^2 = 0$$

$$x^2 + 2x + 1 + y^2 = 0 + 1$$

$$(x+1)^2 + y^2 = 1$$

a. Center: $(-1, 0)$; Radius: $r = 1$

b.



34. $3x^2 + 3y^2 - 12y = 0$

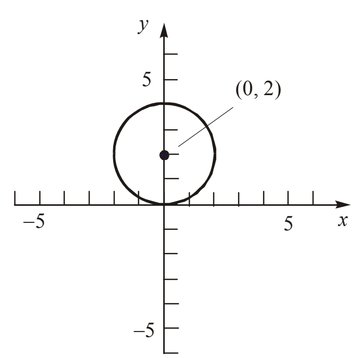
$$x^2 + y^2 - 4y = 0$$

$$x^2 + y^2 - 4y + 4 = 0 + 4$$

$$x^2 + (y-2)^2 = 4$$

a. Center: $(0, 2)$; Radius: $r = 2$

b.



35. Center at (0, 0); containing point (–2, 3).

$$r = \sqrt{(-2-0)^2 + (3-0)^2} = \sqrt{4+9} = \sqrt{13}$$

Equation: $(x-0)^2 + (y-0)^2 = \left(\sqrt{13}\right)^2$

$$x^2 + y^2 = 13$$
$$x^2 + y^2 - 13 = 0$$

36. Center at (1, 0); containing point (–3, 2).

$$r = \sqrt{(-3-1)^2 + (2-0)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

Equation: $(x-1)^2 + (y-0)^2 = \left(\sqrt{20}\right)^2$

$$x^2 - 2x + 1 + y^2 = 20$$
$$x^2 + y^2 - 2x - 19 = 0$$

37. Center at (2, 3); tangent to the x-axis.
$r = 3$

Equation: $(x-2)^2 + (y-3)^2 = 3^2$

$$x^2 - 4x + 4 + y^2 - 6y + 9 = 9$$
$$x^2 + y^2 - 4x - 6y + 4 = 0$$

38. Center at (–3, 1); tangent to the y-axis.
$r = 3$

Equation: $(x+3)^2 + (y-1)^2 = 3^2$

$$x^2 + 6x + 9 + y^2 - 2y + 1 = 9$$
$$x^2 + y^2 + 6x - 2y + 1 = 0$$

39. Endpoints of a diameter are (1, 4) and (–3, 2).
The center is at the midpoint of that diameter:

Center: $\left(\dfrac{1+(-3)}{2}, \dfrac{4+2}{2}\right) = (-1, 3)$

Radius: $r = \sqrt{(1-(-1))^2 + (4-3)^2}$
$= \sqrt{4+1} = \sqrt{5}$

Equation: $(x-(-1))^2 + (y-3)^2 = \left(\sqrt{5}\right)^2$

$$x^2 + 2x + 1 + y^2 - 6y + 9 = 5$$
$$x^2 + y^2 + 2x - 6y + 5 = 0$$

40. Endpoints of a diameter are (4, 3) and (0, 1).
The center is at the midpoint of that diameter:

Center: $\left(\dfrac{4+0}{2}, \dfrac{3+1}{2}\right) = (2, 2)$

Radius: $r = \sqrt{(4-2)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5}$

Equation: $(x-2)^2 + (y-2)^2 = \left(\sqrt{5}\right)^2$

$$x^2 - 4x + 4 + y^2 - 4y + 4 = 5$$
$$x^2 + y^2 - 4x - 4y + 3 = 0$$

41. (c); Center: $(1, -2)$; Radius = 2

42. (d) ; Center: $(-3, 3)$; Radius = 3

43. (b) ; Center: $(-1, 2)$; Radius = 2

44. (a) ; Center: $(-3, 3)$; Radius = 3

45. Let the upper-right corner of the square be the point (x, y). The circle and the square are both centered about the origin. Because of symmetry, we have that $x = y$ at the upper-right corner of the square. Therefore, we get

$$x^2 + y^2 = 9$$
$$x^2 + x^2 = 9$$
$$2x^2 = 9$$
$$x^2 = \frac{9}{2}$$
$$x = \sqrt{\frac{9}{2}} = \frac{3\sqrt{2}}{2}$$

The length of one side of the square is $2x$. Thus, the area is

$$A = s^2 = \left(2 \cdot \frac{3\sqrt{2}}{2}\right)^2 = \left(3\sqrt{2}\right)^2 = 18 \text{ square units.}$$

46. The area of the shaded region is the area of the circle, less the area of the square. Let the upper-right corner of the square be the point (x, y). The circle and the square are both centered about the origin. Because of symmetry, we have that $x = y$ at the upper-right corner of the square. Therefore, we get

$$x^2+y^2=36$$
$$x^2+x^2=36$$
$$2x^2=36$$
$$x^2=18$$
$$x=3\sqrt{2}$$

The length of one side of the square is $2x$. Thus, the area of the square is $\left(2\cdot 3\sqrt{2}\right)^2=72$ square units. From the equation of the circle, we have $r=6$. The area of the circle is $\pi r^2=\pi\left(6\right)^2=36\pi$ square units.

Therefore, the area of the shaded region is $A=36\pi-72$ square units.

47. $x^2+y^2+2x+4y-4091=0$

$$x^2+2x+y^2+4y-4091=0$$
$$x^2+2x+1+y^2+4y+4=4091+5$$
$$\left(x+1\right)^2+\left(y+2\right)^2=4096$$

The circle representing Earth has center $\left(-1,-2\right)$ and radius $=\sqrt{4096}=64$.

So the radius of the satellite's orbit is $64+0.6=64.6$ units.

The equation of the orbit is

$$\left(x+1\right)^2+\left(y+2\right)^2=\left(64.6\right)^2$$
$$x^2+y^2+2x+4y-4168.16=0$$

48. **a.**

$$x^2+(mx+b)^2=r^2$$
$$x^2+m^2x^2+2bmx+b^2=r^2$$
$$(1+m^2)x^2+2bmx+b^2-r^2=0$$

There is one solution if and only if the discriminant is zero.

$$(2bm)^2-4(1+m^2)(b^2-r^2)=0$$
$$4b^2m^2-4b^2+4r^2-4b^2m^2+4m^2r^2=0$$
$$-4b^2+4r^2+4m^2r^2=0$$
$$-b^2+r^2+m^2r^2=0$$
$$r^2(1+m^2)=b^2$$

b. Using the quadratic formula, the result from part (a), and knowing that the discriminant is zero, we get:

$$(1+m^2)x^2+2bmx+b^2-r^2=0$$
$$x=\frac{-2bm}{2(1+m^2)}=\frac{-bm}{\left(\frac{b^2}{r^2}\right)}=\frac{-bmr^2}{b^2}=\frac{-mr^2}{b}$$
$$y=m\left(\frac{-mr^2}{b}\right)+b=\frac{-m^2r^2}{b}+b$$
$$=\frac{-m^2r^2+b^2}{b}=\frac{r^2}{b}$$

c. The slope of the tangent line is m.

The slope of the line joining the point of tangency and the center is:

$$\frac{\left(\frac{r^2}{b}-0\right)}{\left(\frac{-mr^2}{b}-0\right)}=\frac{r^2}{b}\cdot\frac{b}{-mr^2}=-\frac{1}{m}$$

Therefore, the tangent line is perpendicular to the line containing the center of the circle and the point of tangency.

49. $x^2+y^2=9$

Center: (0, 0)

Slope from center to $\left(1, 2\sqrt{2}\right)$ is

$$\frac{2\sqrt{2}-0}{1-0}=\frac{2\sqrt{2}}{1}=2\sqrt{2}\ .$$

Slope of the tangent line is $\frac{-1}{2\sqrt{2}}=-\frac{\sqrt{2}}{4}$.

Equation of the tangent line is:

$$y-2\sqrt{2}=-\frac{\sqrt{2}}{4}\left(x-1\right)$$
$$y-2\sqrt{2}=-\frac{\sqrt{2}}{4}x+\frac{\sqrt{2}}{4}$$
$$4y-8\sqrt{2}=-\sqrt{2}\,x+\sqrt{2}$$
$$\sqrt{2}\,x+4y=9\sqrt{2}$$
$$\sqrt{2}\,x+4y-9\sqrt{2}=0$$

50. $x^2+y^2-4x+6y+4=0$

$$(x^2-4x+4)+(y^2+6y+9)=-4+4+9$$
$$(x-2)^2+(y+3)^2=9$$

Center: (2, –3)

Slope from center to $\left(3, 2\sqrt{2}-3\right)$ is

$$\frac{2\sqrt{2}-3-(-3)}{3-2}=\frac{2\sqrt{2}}{1}=2\sqrt{2}$$

Slope of the tangent line is: $\dfrac{-1}{2\sqrt{2}}=-\dfrac{\sqrt{2}}{4}$

Equation of the tangent line:

$$y-\left(2\sqrt{2}-3\right)=-\frac{\sqrt{2}}{4}(x-3)$$
$$y-2\sqrt{2}+3=-\frac{\sqrt{2}}{4}x+\frac{3\sqrt{2}}{4}$$
$$4y-8\sqrt{2}+12=-\sqrt{2}x+3\sqrt{2}$$
$$\sqrt{2}x+4y-11\sqrt{2}+12=0$$

51. Let (h, k) be the center of the circle.

$$x-2y+4=0\rightarrow 2y=x+4\rightarrow y=\frac{1}{2}x+2$$

The slope of the tangent line is $\dfrac{1}{2}$. The slope from (h, k) to (0, 2) is –2.

$$\frac{2-k}{0-h}=-2 \quad\rightarrow\quad 2-k=2h$$

The other tangent line is $y=2x-7$ and it has slope 2.

The slope from (h, k) to (3, –1) is $-\dfrac{1}{2}$.

$$\frac{-1-k}{3-h}=-\frac{1}{2}$$
$$2+2k=3-h$$
$$2k=1-h$$
$$h=1-2k$$

Solve the two equations in h and k:

$$2-k=2(1-2k)$$
$$2-k=2-4k$$
$$3k=0$$
$$k=0$$
$$h=1-2(0)=1$$

The center of the circle is (1, 0).

52. Find the centers of the two circles:

$$x^2+y^2-4x+6y+4=0$$
$$(x^2-4x+4)+(y^2+6y+9)=-4+4+9$$
$$(x-2)^2+(y+3)^2=9$$

Center: $\left(2,-3\right)$

$$x^2+y^2+6x+4y+9=0$$
$$(x^2+6x+9)+(y^2+4y+4)=-9+9+4$$
$$(x+3)^2+(y+2)^2=4$$

Center: $\left(-3,-2\right)$

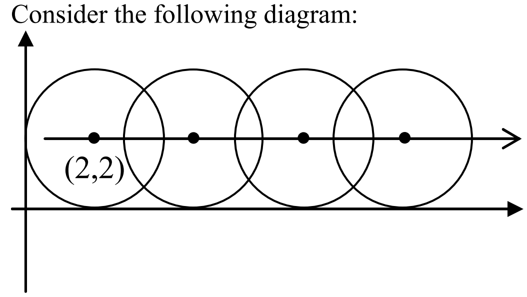
Find the slope of the line containing the centers:

$$m=\frac{-2-(-3)}{-3-2}=-\frac{1}{5}$$

Find the equation of the line containing the centers:

$$y+3=-\frac{1}{5}(x-2)$$
$$5y+15=-x+2$$
$$x+5y=-13$$
$$x+5y+13=0$$

53. Consider the following diagram:



Therefore, the path of the center of the circle has the equation $y=2$.

54.
$$C=2\pi r$$
$$6\pi=2\pi r$$
$$\frac{6\pi}{2\pi}=\frac{2\pi r}{2\pi}$$
$$3=r$$

The radius is 3 units long.

55. (b), (c), (e) and (g)
We need $h,k>0$ and $\left(0,0\right)$ on the graph.

56. (b), (e) and (g)
We need $h<0$, $k=0$, and $|h|>r$.

57. Answers will vary.